Solar Photovoltaics: Principles, Technologies & Materials
Prof. Ashish Garg
Department of Material Science & Engineering
Indian Institute of Technology, Kanpur

Lecture – 05
Sun-Earth Angular Relations

Welcome to lecture 5, of the course Solar Photovoltaics Principles Technologies and Materials.

Just a brief recap.

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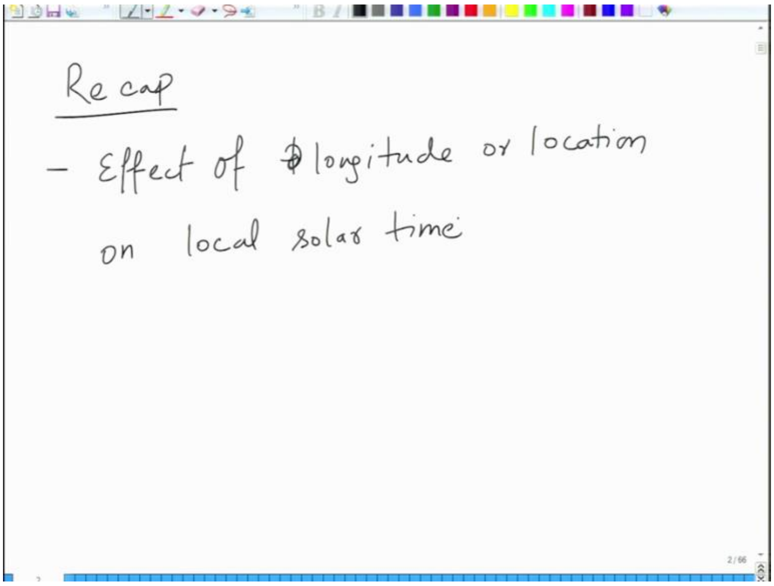

So, in the last lecture we looked at effect of , longitude or location on local solar times. Essentially, we looked at a few expressions, and since we mean you can understand this very well because for example, for this country we calculate the time which is 530 hours ahead of GMT, ok. But India is a vast country and the longitude varies by at least 5 to 10 degrees, as a result time in different locations cannot be same, hence the solar noon will not be the same.

So, for example, if you go to far East of the country the sun rises earlier whereas, if you go to the Western part sun rises later whereas, the clocks have set at the same time. So, as a result there is a difference between the time when the sun is at zenith that is actual noon with respect to the noon, that is shown by your watch. So, to correct for these differences we need to look at certain expressions by which you can calculate the time correction which will correct the solar time which will help you to calculate the actual time at which the sun will be at the zenith. So, that will be useful from the perspective of calculation of solar radiation intensity whose details we will see later on.

So, now, we will look at some more details of geometrical relationship between earth and sun.

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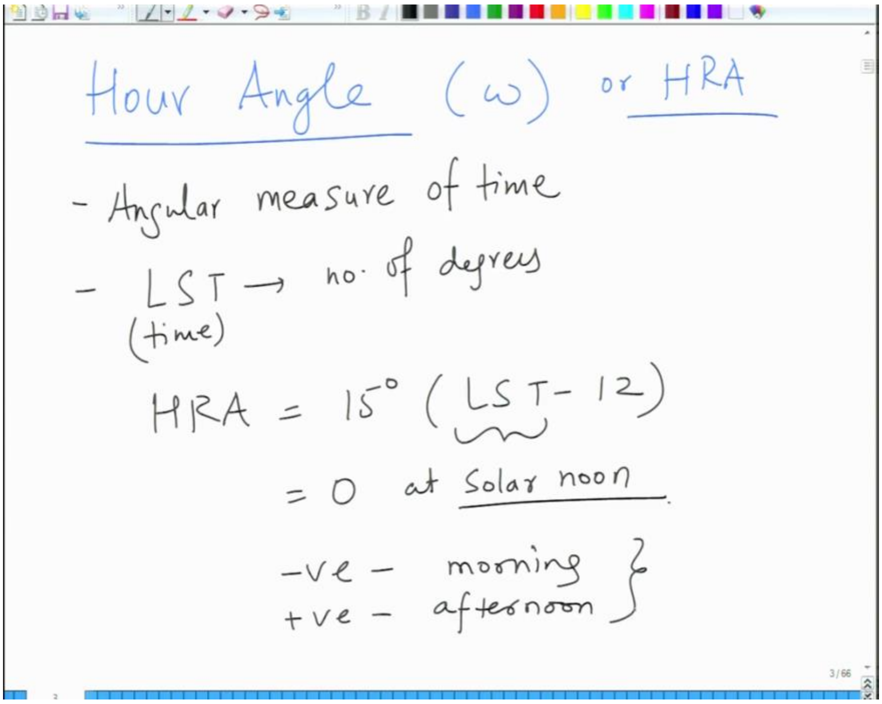

$$Hour\ angle\ \omega\ or\ HRA$$

$$-\ Angular\ measure\ of\ time$$

$$-\ LST --> no of degrees$$

$$HRA = 15^{O}\ (LST - 12)$$

$$= 0\ at\ solar\ noon$$

$$-ve --> morning$$

$$+ve --> afternoon$$

So, we will first look at a quantity called as Hour Angle which is called as omega and this is depicted as HRA. So, HRA is essentially angular measure of time, and it basically converts, LST into number of degrees by which the sun moves through the sky. So, HRA is depicted as 15 degrees into LST minus 12. So, by definition our angle is 0 at solar noon. So, of course, when you have solar noon then LST is equal to 12 and which means this is 0 degree at solar noon. So, you can see that in the morning this is negative in morning and positive in afternoon.

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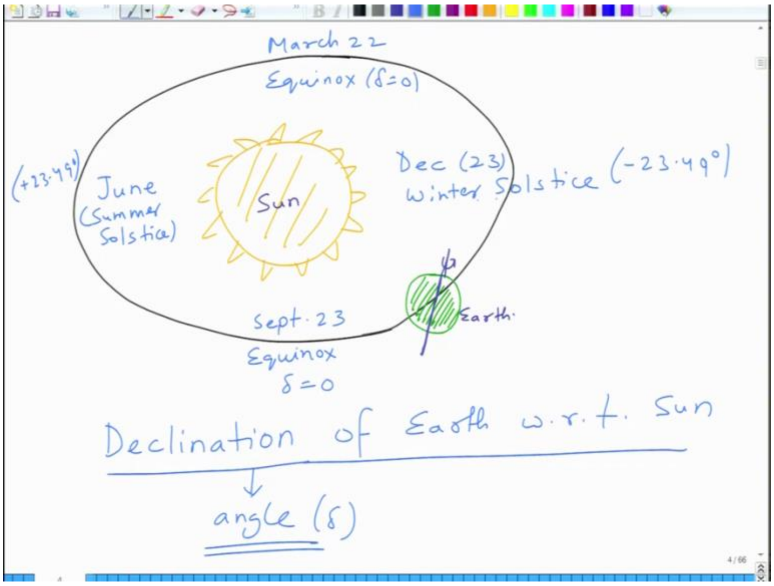

If you look at the rotation of earth around the sun, so this is our sun, and around which you have the orbit in which the earth moves and this is our earth. And from earth there is something called as an axis, this is this is axis of fault, axis of rotation of earth around which it is rotating. So, this is sun, this is earth.

So, there is certain thing called as declination angle, because this axis is not parallel to the orbital axis as a result you have weather changes. And this is depicted in the form of what we call as declination angle. So, this gives rise to an angle called as declination angle. And so, earth goes through various locations, so around at this point let us say this is June and this is December. So, around December 23 you have winter solstice, and June you have summer solstice. What it means is that the declination angle is minimum that is it goes through a value of minus 23.49 degrees in winter and summer it goes through minus plus

23.49. So, we are taking this for Northern hemisphere. And then you have March 22 you have a equinox when this angle delta is equal to 0. So, this is this angle is called a delta.

And then you have September 23, when you have a equinox, then again delta is equal to 0, what it means is that because of this axis of earth rotation being tilted you have shortest day that is in December which would mean for shortest day for Northern hemisphere whereas, it would be long as they for Southern hemisphere. You have March 22 which is equinox which means day and night are equal and that is when this lambda is equal to 0, we will explain to you what lambda is in a minute and then we have June's summer solstice when the day is the longest, and then we have September 23 when the day and night were again equal in length.

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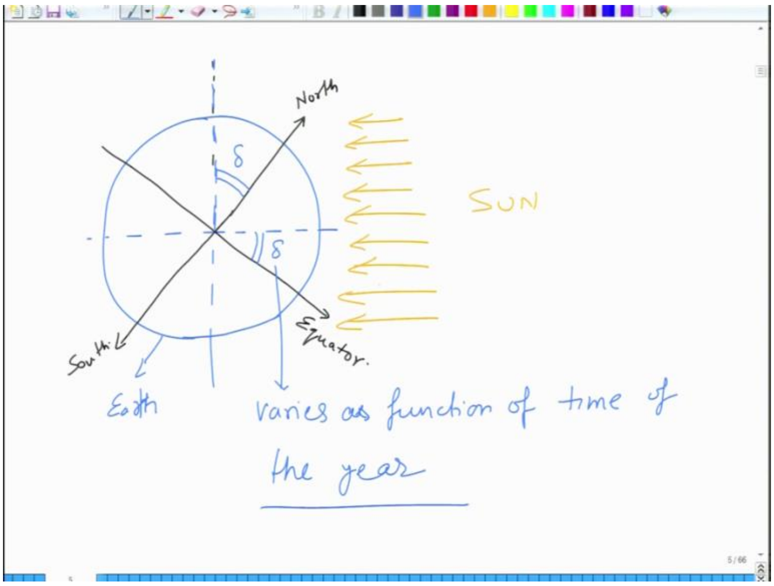

So, because of this centricity it goes through these seasonal variations. And the way we calculate this angle as, so we have this earth. Let us say now we do or we draw it with respect to earth. So, this is earth.

This is the vertical axis, and let us say sunlight is coming from this direction. So, this is sunray. And this is let us say, the North and this would be South, North, South axis and this is where we have equator. So, this is equator. So, here this angle whatever we see here with respect to, with respect to this vertical this angle is called as that declination angle. Or we can say since this angle is delta, the angle between the horizontal and the equator is parallel to the sun beam. So, angle which is made by sunray with respect to the equator is called as declination angle, delta.

And this changes because North South undergoes a rotation because this does not remain in the same position all the time, this delta also changes as a function of time. So, this delta varies as a function of time of the year because earth is also changing throughout the year.

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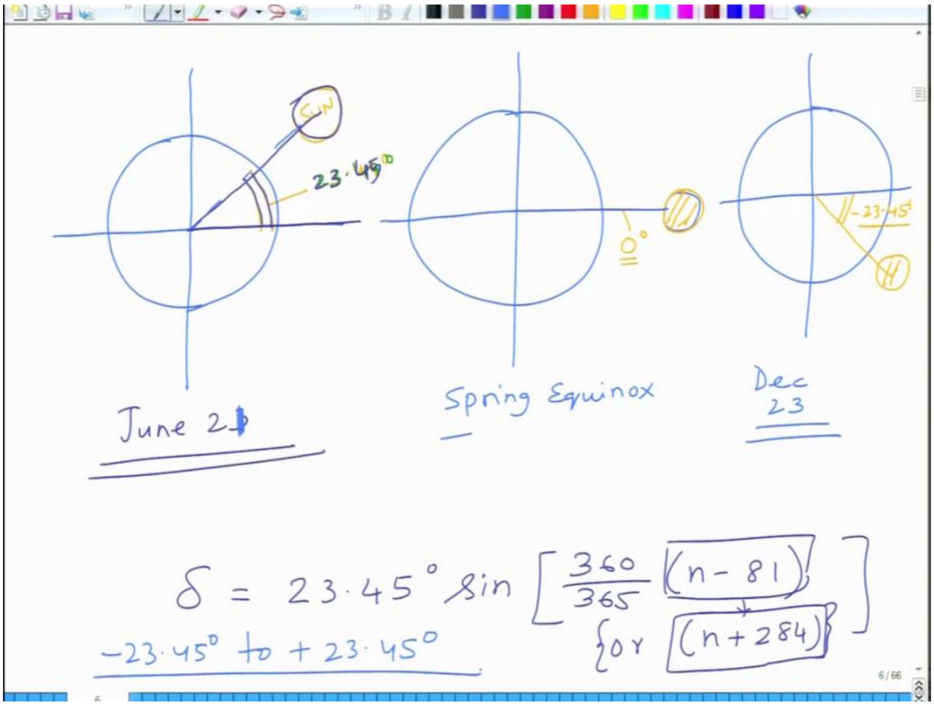

$$\delta = 23.45^0 \ sin[360 \ ((n-81)or(n+284))/365]$$
$$\delta \ lies \ between \ -23.45^0 \ and \ +23.45^0$$

So, let us say if we again plot earth, so in the summer solstice this is the sun. So, this is the declination which is 23.49 degree we will give the formula in a little while or maybe we can write the formula just now. So, the way you calculate this declination angle is worked out, this declination angle is defined as delta is equal to 23.45 degrees into sorry its 45 not 49 sine of 360 divided by 365 into n minus 81 or you can also write it as n plus 284 because if you sum them up its nothing but 365. So, this is the expression. So, you can substitute n minus 81 by n plus 284 is to get the same value.

So, declination angle is basically an angle which is made by joining the center of the sun and the earth with its projection on the equatorial plane. So, when you look at June 23 in summer solstice the sun so, this is the angle between the sun beam and the equatorial plane, ok. So, this is 23.45 the maximum it can achieve. If you look at now other position. So, other position would be, all right. So if this is sun then the angle is equal to 0 degree which would be during (Refer Time: 11:14).

Spring equinox. And if you take it to the other side it will be the other equinoxes march equinox 23,

It is June 23; just one second let me check. June 21, you write here its June 21, there is June 21 here and again it is June 21 here, and December is 23. And then we go for the winter one. So, this is earth, sun and for the winter one you would have sun somewhere here, and this will do the angle which is minus 23.45 degree. So, this would be December 23. So, this is winter solstice, this is a summer solstice, this is equinox.

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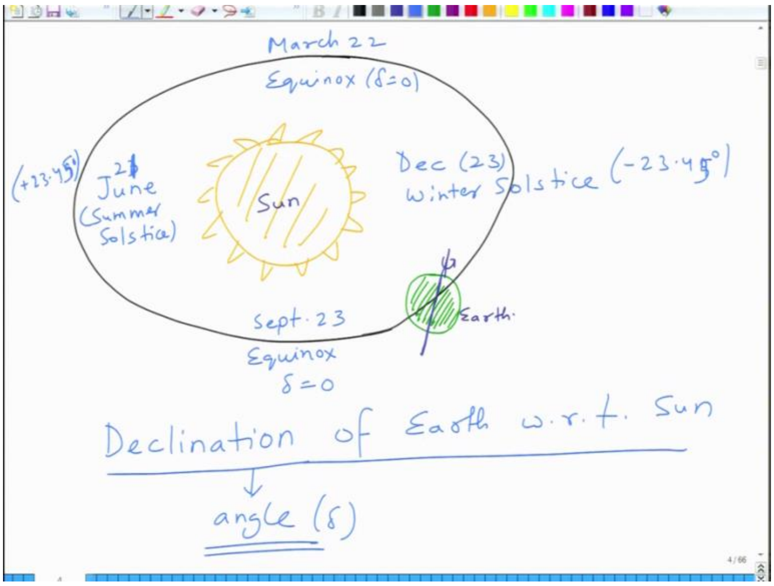

And let me just correct the values this is 45, ok. So, the first angle that we have worked out is delta which is a declination angle is 23.45 degrees multiplied by sine of 360 divided by 365 into n minus 81. So, naturally you can see the value changes form plus minus 23.45 to plus 23.45 degrees as a function of day of the year.

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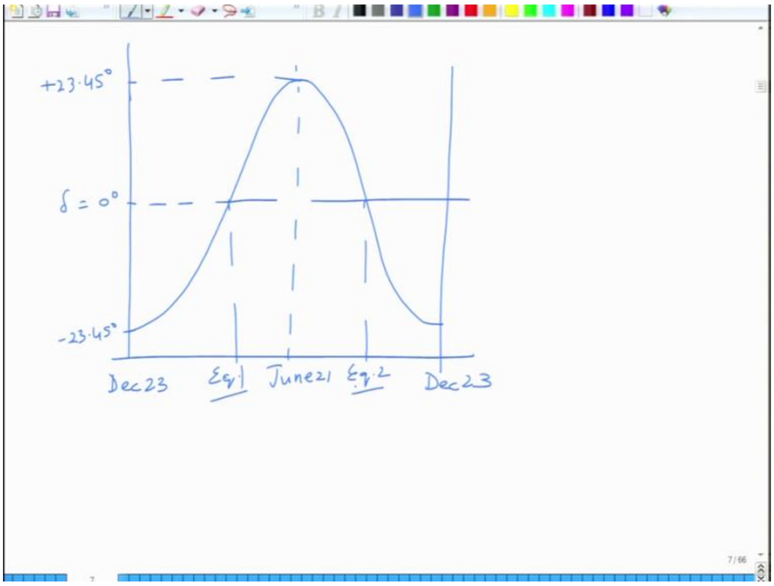

So, if you plot the values, so let us even we plot the values from December 23 to December 23 let us say so, this goes from somewhere here we will have June 21. So, this will go from minus 23.45 to plus 23.45 back to minus 23.45. So, this would be plus 23.45 degrees. This is how it will vary throughout the year and somewhere in between it will go through 0 degrees. So, you will have two times. So, these will be your equinoxes when delta will be equal to 0 degrees. So, equinox 1 and this will be equinox 2 in March and September. And then we define some more angles.

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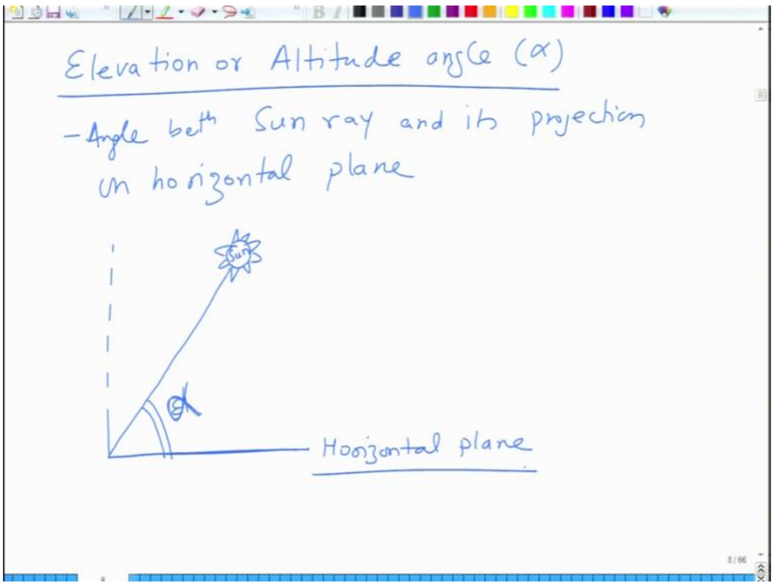

So, the first angle that we define is called as elevation angle or altitude angle, which is called an alpha. This angle is defined as the angle between some ray and its projection on horizontal plane. So, let us say this is the vertical this is the horizontal plane, this is sun, this is the sunbeam that is coming. So, this is the value which is called as elevation. So, you can say this is alpha. And then you calculate, so this is let us say horizontal plane (Refer Time: 15:36).

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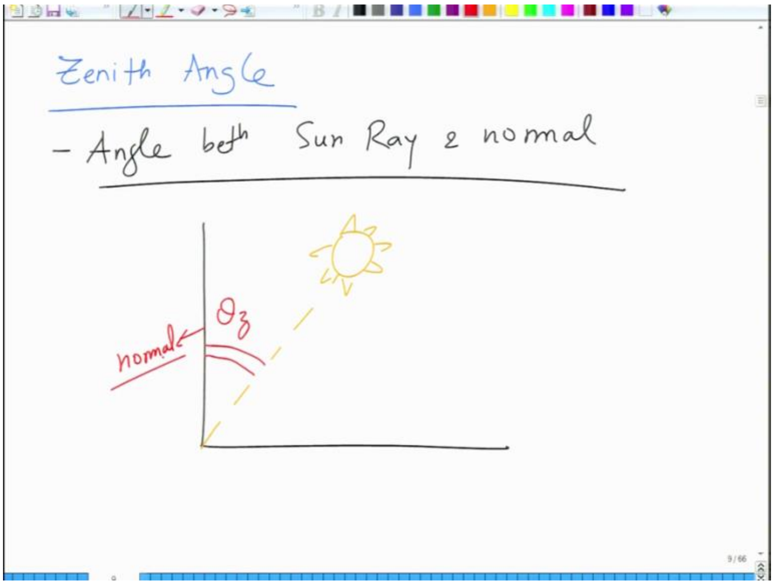

Next angle that we determine is zenith angle. Zenith angle is the angle between sunray and a normal. So, essentially this is the normal, this is the horizontal, this is the sun and this is a sunbeam, the angle here is called as theta z, this is normal.

Under certain conditions theta z plus alpha will become equal to 90 degrees, but they will not be 90 degrees for all the conditions because remember the sun will sun always move from North to South in the sky. So, those two angles will always not match equal to 90 degrees that sum, but under certain condition they will become or equal to 90 degrees.

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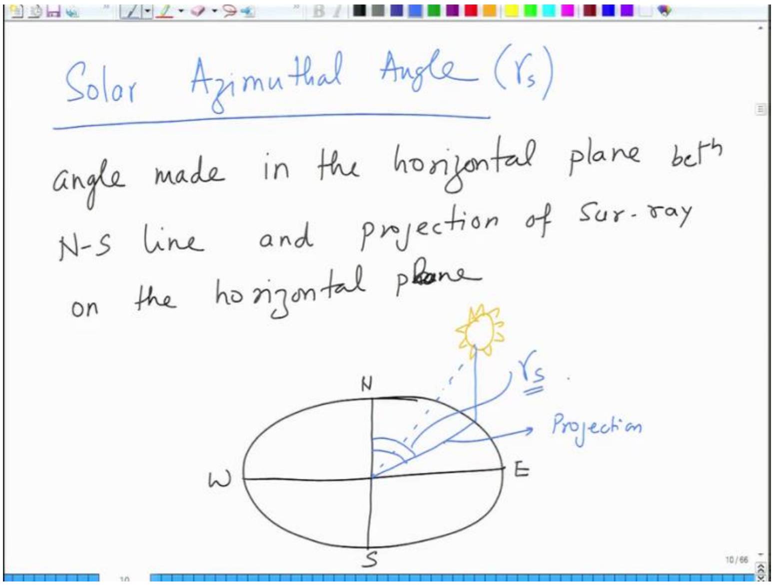

And then we have third angle called as solar azimuthal angle. The angle which is in the horizontal plane between North South line and projection of sun ray on the horizontal plane. Let me just draw it for you. So, if this is the horizontal plane, all right and if I find it find this North this is South East this is West, let us say sun is somewhere here.

So, this is sun and, so this is the sun ray right this is the projection. So, this is projection. This is the North South line, this is the angle which is gamma. The solar is azimuthal angle. So, you can see naturally the solar azimuthal angle. So, if you are going from East to West. So, if it is an East it goes to plus 90 degrees and if you go to West it goes to minus 90 degrees.

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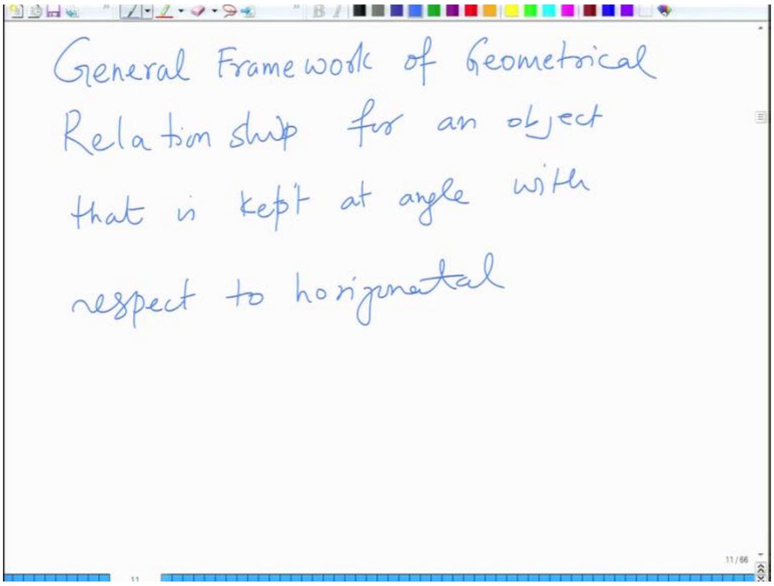

So, basically thusing these angles we can frame a general framework, so we can make a general framework, of geometrical relationship for an object that is kept at an angle with respect to the horizontal. So, most of things that we have seen so far are for horizontal surface, but the solar panel on the surface is kept itself for a certain angle. So, we need to also bring that angle into account.

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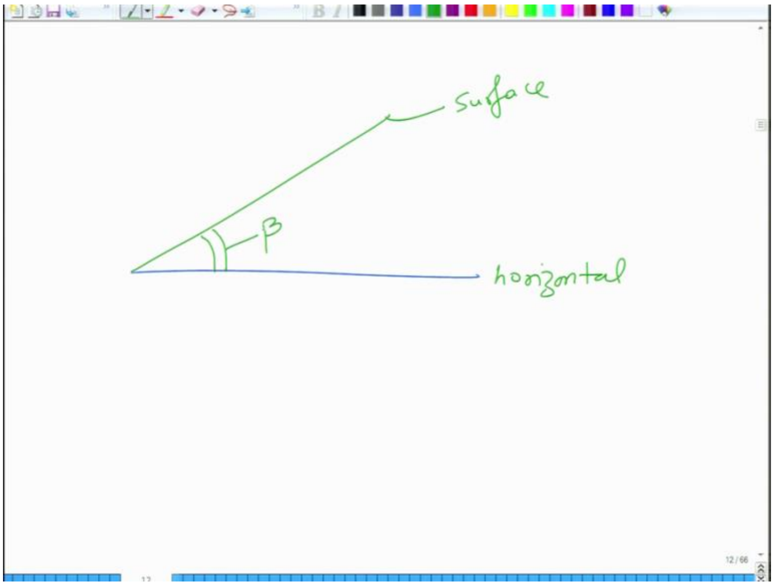

So, generally what will happen is that, this is your horizontal and somewhere here you will have inclined surface. So, this is your surface which receives the radiation and this is your horizontal. So, that self makes an angle called as beta. So, this is what we need to see. So, let us see a PPT look for those relationships.

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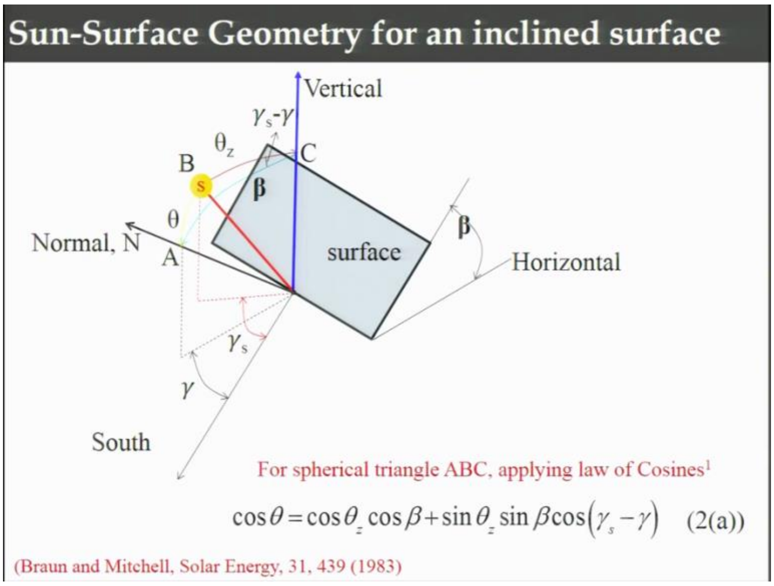

$$For\ spherical\ triangle\ ABC\ applying\ law\ of\ cosines$$
$$cos\ \theta = cos\theta_z\ cos\beta + \ sin\theta_z\ sin\ \beta\ cos(\gamma_s\ \ - \gamma) - - - - - - - - - - (2(a))$$

So, this is basically a figure which shows the relationship of a surface let us say a solar panel which is inclined with respect to horizontal at an angle beta. So, essentially, we are going to look at the relationship between various angles for inclined surface. So, sun surface geometry for inclined surface. So, this grey one is the surface which is inclined at an angle beta with respect to the South and is pointing in the direction which is shown by this arrow.

Now, detailed analysis of this whole thing you can see in this reference which we have noted here. So, its Braun and Mitchell, Solar Energy, it is a paper, which got published in 1983 and this has a detailed geometrical analysis of these relations. So, unfortunately, we do not have enough time to go through these, but we will just look at various segments of it, key segments.

So, this is the vertical with respect to the horizontal surface. And this is the sun. So, sun beam is the red one which is coming for a given point. So, this angle which is between the sun ray and the vertical is theta z, and this is the normal to the surface. Normal to the surface does not necessarily mean the sun beam, you can have a normal to the surface at a different angle. So, this angle which is the normal to the surface and the sun is the theta angle, angle theta, and this is what we are interested in calculating.

The angle between the sun beam and the normal to the surface. So, I can first calculate what is the solar azimuthal angle. Solar azimuthal or angle is the angle between projection of sun beam on the horizontal and the North South direction which is gamma s. and another is azimuthal angle which is the projection of North of the surface on the North South plane, so this is normal to the surface, if we take its projection on the North South plane and then calculate its angle with respect to the North South axis in the horizontal plane then this angle is gamma. And then there is a angle which is the angle between the vertical and to the horizontal and the normal to the surface, this angle is gamma s minus gamma that can be shown by geometry.

So, if I say that this point is A, this point is B, this point is C then this makes a spherical triangle and we know various angles between these. If we know these angles and we consider this ABC as a spherical triangle, then if we apply the law of cosines to this spherical triangle then first thing that we work out is cos theta is equal to cos theta z into cos theta cos beta plus sine theta z sine beta into cos gamma s minus gamma that is the first relation that I get for cos theta, all right.

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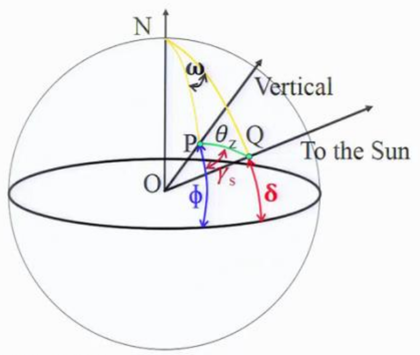

P: point of observation at hour angle w
Q: Sun normal point
O: Center of the Earth

Law of Cosines for NPQ spherical triangle

$$\cos\theta_z = \sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega \quad (2(b))$$

Law of Sines for NPQ gives

$$\sin\gamma_s = \frac{\sin\omega\cos\delta}{\sin\theta_z} \quad (2(c))$$

$$Law\ of\ cosines\ for\ NPQ\ spherical\ triangle$$
$$cos\theta_z = \ sin\delta\ sin\phi + \ cos\delta\ cos\phi\ cos\omega ----------(2(b)$$
$$Law\ of\ sines\ for\ NPQ\ gives$$
$$sin\gamma_s = sin\omega\ cos\delta / sin\theta_z --------(2(c))$$

And then we have this another figure in which we can now correlate these angles in a little different form. So, what we have done is here, we have now taken it with respect to earth. So, O is the centre of the earth, this is the direction which is to the sun. So, this is basically the beam. This is the vertical which is with respect to the horizontal surface, and this is a vertical which is with respect to horizontal surface. Horizontal surface does not necessarily need to be, see horizontal does not mean equator like equator and the horizontal are two different things for certain location horizontal will be equal to equator. But for certain other like location for example, if you are sitting in china the horizontal is not the equator, but if you have a country which is located on the equator then horizontal is equal to equator. So, horizontal and equator does not necessarily mean the same thing it is location dependent. So, as a result the vertical is not necessarily the North axis.

As we saw in this figure earlier this is South, North is in that direction and North and the vertical do not necessarily coincide that is dependent upon the location. Similarly, that is what I am trying to show in this location. So, if you do that if you represent the same figure with respect to earth then we have this, so this is North of the earth, on this side you will have South, this is the vertical with respect to horizontal, this is a sunray and then again you can make a triangle which is N, P and Q. So, so this angle between NP and NQ is the hour angle, HRA and the angle between P and Q is does anything angle because sunray and the vertical are the they constitute theta z.

And similarly, if you look at this P location, P is the location which is essentially the location at which you are with respect to equator. So, this is nothing but your latitude. So, this is latitude. So, this latitude is the angle between equator and the point P. Gamma s is the solar azimuthal angle which is the angle between the sun beam its position the horizontal plane with respect to the North South. So, gamma s says this is North South and this is sort of you can say one great circle, this is you can say another great circle so, essentially the angle between these two, so, this green line is the one which you can say, so this is P and Q, P is the location which depicts the vertical and Q is the direction of the sun.

So, if you again go back to previous figure, just try to correlate this. Gamma s is the angle between sun beam projection this is that projection with respect to North South. So, this is the vertical axis North South of earth's axis and we can say horizontal not earth's axis, but with respect to the horizontal. So, this is North South and then you have this projection and the angle between these two is gamma s. So, this is the angle which is represented as angle between these two points the green line P Q and this latitude.

So, this basically is with respect to earth's position and this is the position which we can say the sun beam and its projection on the North South plane. So, the angle between those two is gamma s. So, if we apply again, we can see that P is the point of observation at our angle omega, Q is the sun normal point and O is the center of the earth. So, for NPQ again when we apply law of cosines we get relation for cos theta z and this cos theta z turns out to be sine delta sine phi plus cos delta cos phi cos omega.

And again, when we apply law of sines for N P Q we get sine gamma s is equal to sine omega cos delta divided by sine theta. So, we do not have time to get into the proofs of these, but if you want to get the details of the proof you can refer to this paper which I referred earlier. So, these 3 equations when they are combined for various conditions, they give you certain geometrical relationships.

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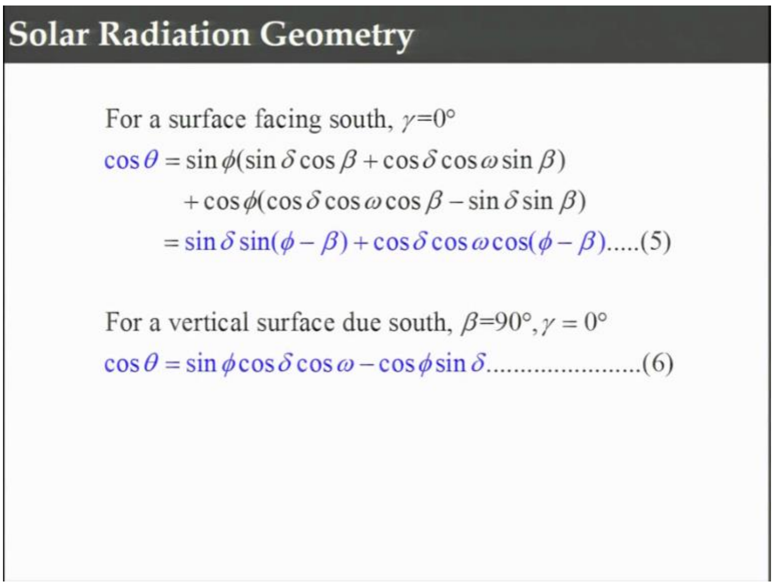

$$For\ a\ surface\ facing\ south,\ \gamma = 0^0$$

$$cos\theta = \ sin\phi(sin\delta\ cos\beta + \ cos\delta\ cos\omega\ sin\beta) + \ cos\phi(cos\delta\ cos\omega\ cos\beta - sin\delta\ sin\beta)$$

$$= sin\delta\ sin(\phi - \ \beta) + \ cos\delta\ cos\omega\ cos(\phi - \ \beta)............(5)$$

$$For\ a\ vertical\ surface\ due\ south,\ \beta = 90^0,\ \gamma = 0^0$$

$$cos\theta = sin\phi\ cos\delta\ cos\omega - \ cos\phi\ sin\delta$$

So, for example, to simplify the matters, if we have a vertical surface which means beta is equal to 90 degree, then cos theta can be simplified by combining the above 3 equations as sine phi cos delta cos gamma cos omega minus cos phi sine delta cos gamma sine beta plus cos delta sine gamma sine omega that is the first relation that gets modified for a vertical surface.

Similarly, for a horizontal surface, which means beta is equal to 0, then again, this equation gets modified and this becomes very simple equation in that case cos theta is equal to sine phi sine delta. For a horizontal surface theta is theta z, so theta gets modified, so this theta is nothing but theta r in that case cos phi cos delta and cos omega.

So, likewise we can do various things for example, we can have a surface facing South. If surface itself is facing South which means gamma is equal to 0 because then the projection of vertical coincides with a North South as a result gamma is equal to 0. So, cos theta then again gets modified to sine delta and to sine phi minus beta plus cos delta cos omega into cos phi minus beta. And if we have a vertical face which is due South then again it gets modified to simpler equation. So, those equations are complex, but by looking at these simple variations or simplistic expressions we can modify those expressions to much simpler forms.

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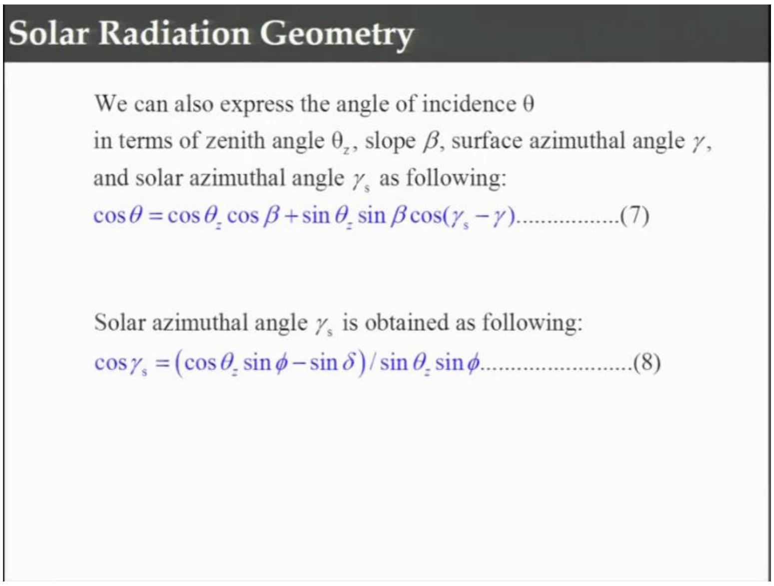

Solar Radiation Geometry

$$cos\theta = cos\theta_z \; cos\beta + \; sin\theta_z \; sin\beta \; cos(\gamma_s - \gamma)..........(7)$$

$$Solar\ azimuthal\ angle\ \gamma_s,\ is\ obtained\ as\ following:$$
$$cos\gamma_s = (cos\theta_z \; sin\phi - sin\delta)/sin\theta_z \; .sin\phi..............(8)$$

So, we will cut here. I think we have run out of time. The remaining exercise of this geometrical relationship we can do in the next class, ok.