Economic Order Quantity

Read this description of EOQ, which will help you understand the fundamental function of this equation. EOQ is important because it helps minimize the total holding and ordering costs related to inventory. Pay close attention to when this applies in the production process.

In inventory management, **economic order quantity** (**EOQ**) is the order quantity that minimizes the total holding costs and ordering costs. It is one of the oldest classical production scheduling models. The model was developed by Ford W. Harris in 1913, but R. H. Wilson, a consultant who applied it extensively, and K. Andler are given credit for their in-depth analysis.

Overview

EOQ applies only when demand for a product is constant over the year and each new order is delivered in full when inventory reaches zero. There is a fixed cost for each order placed, regardless of the number of units ordered. There is also a cost for each unit held in storage, commonly known as holding cost, sometimes expressed as a percentage of the purchase cost of the item.

We want to determine the optimal number of units to order so that we minimize the total cost associated with the purchase, delivery and storage of the product.

The required parameters to the solution are the total demand for the year, the purchase cost for each item, the fixed cost to place the order and the storage cost for each item per year. Note that the number of times an order is placed will also affect the total cost, though this number can be determined from the other parameters.

Variables

* TCTC = total annual inventory cost
* PP = purchase unit price, unit production cost
* QQ = order quantity.
* Q∗Q∗ = optimal order quantity.
* DD = annual demand quantity.
* KK = fixed cost per order, setup cost (*not* per unit, typically cost of ordering and shipping and handling. This is not the cost of goods)
* hh = annual holding cost per unit, also known as carrying cost or storage cost (capital cost, warehouse space, refrigeration, insurance, etc. usually not related to the unit production cost)

The total cost function and derivation of EOQ formula

The single-item EOQ formula finds the minimum point of the following cost function:

Total Cost = purchase cost or production cost + ordering cost + holding cost

Where:

* Purchase cost: This is the variable cost of goods: purchase unit price × annual demand quantity. This is P × D
* Ordering cost: This is the cost of placing orders: each order has a fixed cost K, and we need to order D/Q times per year. This is K × D/Q
* Holding cost: the average quantity in stock (between fully replenished and empty) is Q/2, so this cost is h × Q/2

TC=PD+DKQ+hQ2TC=PD+DKQ+hQ2

To determine the minimum point of the total cost curve, calculate the derivative of the total cost with respect to Q (assume all other variables are constant) and set it equal to 0:

0=−DKQ2+h20=−DKQ2+h2

Solving for Q gives Q\* (the optimal order quantity):

Q∗2=2DKhQ∗2=2DKh

Therefore:

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Q∗=2DKh−−−−√Q∗=2DKh

Q\* is independent of P; it is a function of only K, D, h.

The optimal value Q\* may also be found by recognising that

TC=DKQ+hQ2+PD=h2Q(Q−2DK/h−−−−−−√)2+2hDK−−−−−√+PDTC=DKQ+hQ2+PD=h2Q(Q−2DK/h)2+2hDK+PD, where the non-negative quadratic term disappears for Q=2DK/h−−−−−−√Q=2DK/h, which provides the cost minimum TCmin=2hDK−−−−−√+PDTCmin=2hDK+PD.

Example

* annual requirement quantity (D) = 10000 units
* Cost per order (K) = 40
* Cost per unit (P)= 50
* Yearly carrying cost per unit (h) = 5

Economic order quantity = 2D∗Kh−−−−√2D∗Kh =2∗10000∗405−−−−−−−√=2∗10000∗405 = 400 units

Number of orders per year (based on EOQ) =10000400=2510000400=25

Total cost =P∗D+K(D/EOQ)+h(EOQ/2)=P∗D+K(D/EOQ)+h(EOQ/2)

Total cost =50∗10000+40∗(10000/400)+5∗(400/2)=502000=50∗10000+40∗(10000/400)+5∗(400/2)=502000

If we check the total cost for any order quantity other than 400(=EOQ), we will see that the cost is higher. For instance, supposing 500 units per order, then

Total cost =50∗10000+40∗(10000/500)+5∗(500/2)=502050=50∗10000+40∗(10000/500)+5∗(500/2)=502050

Similarly, if we choose 300 for the order quantity then

Total cost =50∗10000+40∗(10000/300)+5∗(300/2)=502083.33=50∗10000+40∗(10000/300)+5∗(300/2)=502083.33

This illustrates that the economic order quantity is always in the best interests of the firm.

Extensions of the EOQ model

Quantity discounts

An important extension to the EOQ model is to accommodate quantity discounts. There are two main types of quantity discounts: (1) all-units and (2) incremental. Here is a numerical example:

* Incremental unit discount: Units 1–100 cost $30 each; Units 101–199 cost $28 each; Units 200 and up cost $26 each. So when 150 units are ordered, the total cost is $30\*100 + $28\*50.
* All units discount: an order of 1–1000 units costs $50 each; an order of 1001–5000 units costs $45 each; an order of more than 5000 units costs $40 each. So when 1500 units are ordered, the total cost is $45\*1500.

In order to find the optimal order quantity under different quantity discount schemes, one should use algorithms; these algorithms are developed under the assumption that the EOQ policy is still optimal with quantity discounts. Perera et al. (2017) establish this optimality and fully characterize the (s,S) optimality within the EOQ setting under general cost structures.

Design of optimal quantity discount schedules

In presence of a strategic customer, who responds optimally to discount schedule, the design of optimal quantity discount scheme by the supplier is complex and has to be done carefully. This is particularly so when the demand at the customer is itself uncertain. An interesting effect called the "reverse bullwhip" takes place where an increase in consumer demand uncertainty actually reduces order quantity uncertainty at the supplier.

Backordering costs and multiple items

Several extensions can be made to the EOQ model, including backordering costs and multiple items. Additionally, the economic order interval can be determined from the EOQ and the economic production quantity model (which determines the optimal production quantity) can be determined in a similar fashion.

A version of the model, the Baumol-Tobin model, has also been used to determine the money demand function, where a person's holdings of money balances can be seen in a way parallel to a firm's holdings of inventory.

Malakooti (2013) has introduced the multi-criteria EOQ models where the criteria could be minimizing the total cost, Order quantity (inventory), and Shortages.

A version taking the time-value of money into account was developed by Trippi and Lewin.

Imperfect quality

Another important extension of EOQ model is to consider items with imperfect quality. Salameh and Jaber (2000) are the first to study the imperfect items in an EOQ model very thoroughly. They consider an inventory problem in which

the demand is deterministic and there is a fraction of imperfect items in the lot and are screened by the buyer and sold by them at the end of the circle at discount price. Imperfect quality items have also been considered in a decentralized supply chain and the problem has also been studied with game theoretical models.

For improving fuel economy of internal combustion engines

Recently an interesting similarity between EOQ of Melon picking and fuel injection in Gasoline Direction Injection has been proposed.

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