Statistical Process Control

Statistical Monitoring of Processes to Detect Special Causes of Variation

Lecture Outline

- Basics of Statistical Process Control
- Control Charts
- Control Charts for Attributes
- Control Charts for Variables
- Control Chart Patterns
- SPC with Excel
- Process Capability

Statistical QA Approaches

Statistical process control (SPC)

- Monitors production process to prevent poor quality

Acceptance sampling

- Inspects random sample of product to determine if a lot is acceptable

Design of Experiments

Statistical Quality Assurance

Purpose: Assure that processes are performing in an acceptable manner

Methodology: Monitor process output using statistical techniques

If results are acceptable, no further action is required
Unacceptable results call for corrective action

Acceptance Sampling:
Quality assurance that relies primarily on inspection before and after production

Statistical Process Control (SPC):
Quality control efforts that occur during production

What is SPC?

A simple, yet powerful, collection of tools for graphically analyzing process data

Has one primary purpose: To tell you when you have a problem

Invented by Walter Shewhart at AT&T to minimize process tampering

Important because unnecessary process changes increase instability and increase the error rate

SPC will identify when a problem (or special cause variation) occurs

Basics of Statistical Process Control

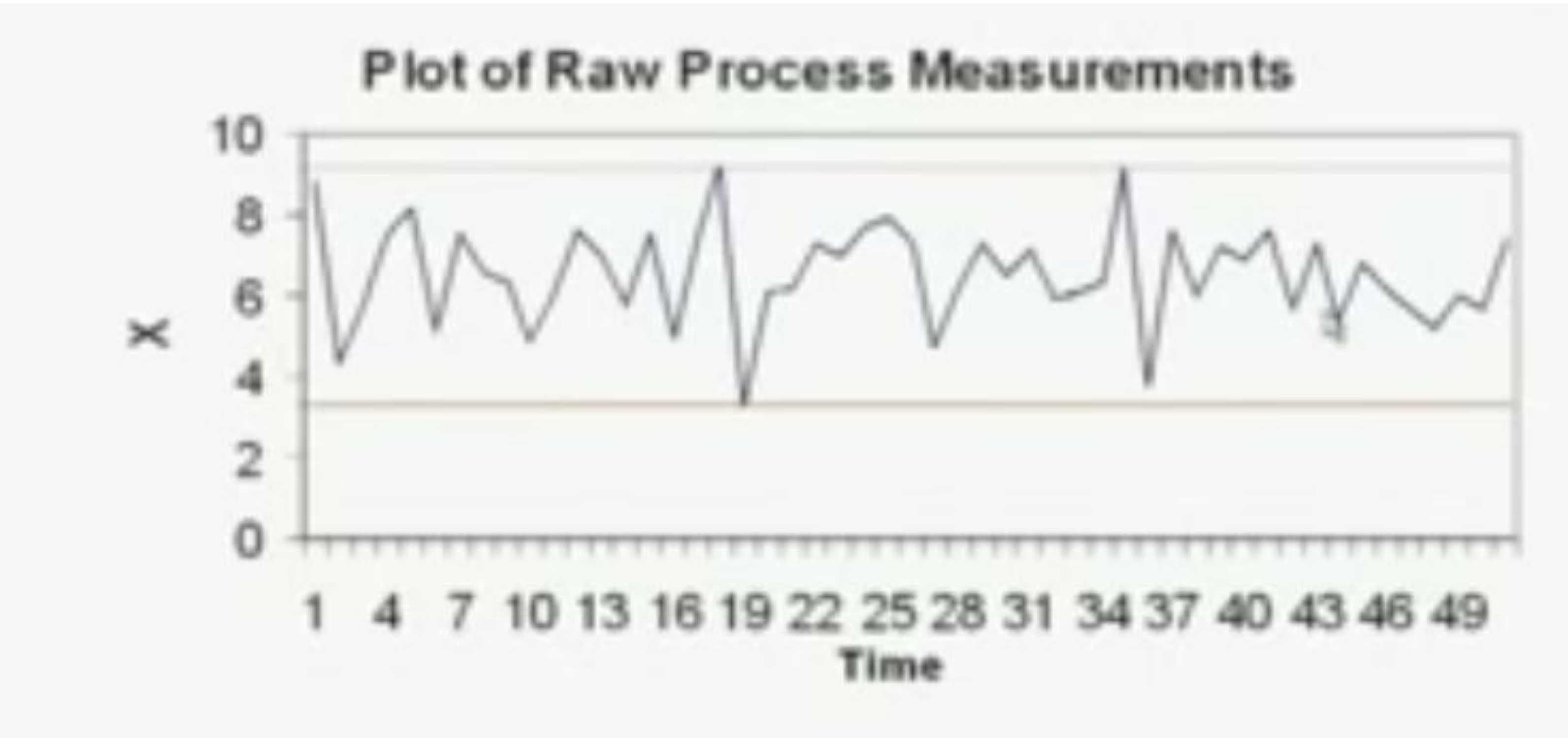
Statistical Process Control (SPC)
- Monitoring production process to detect and prevent poor quality

Sample
- Subset of items produced to use for inspection

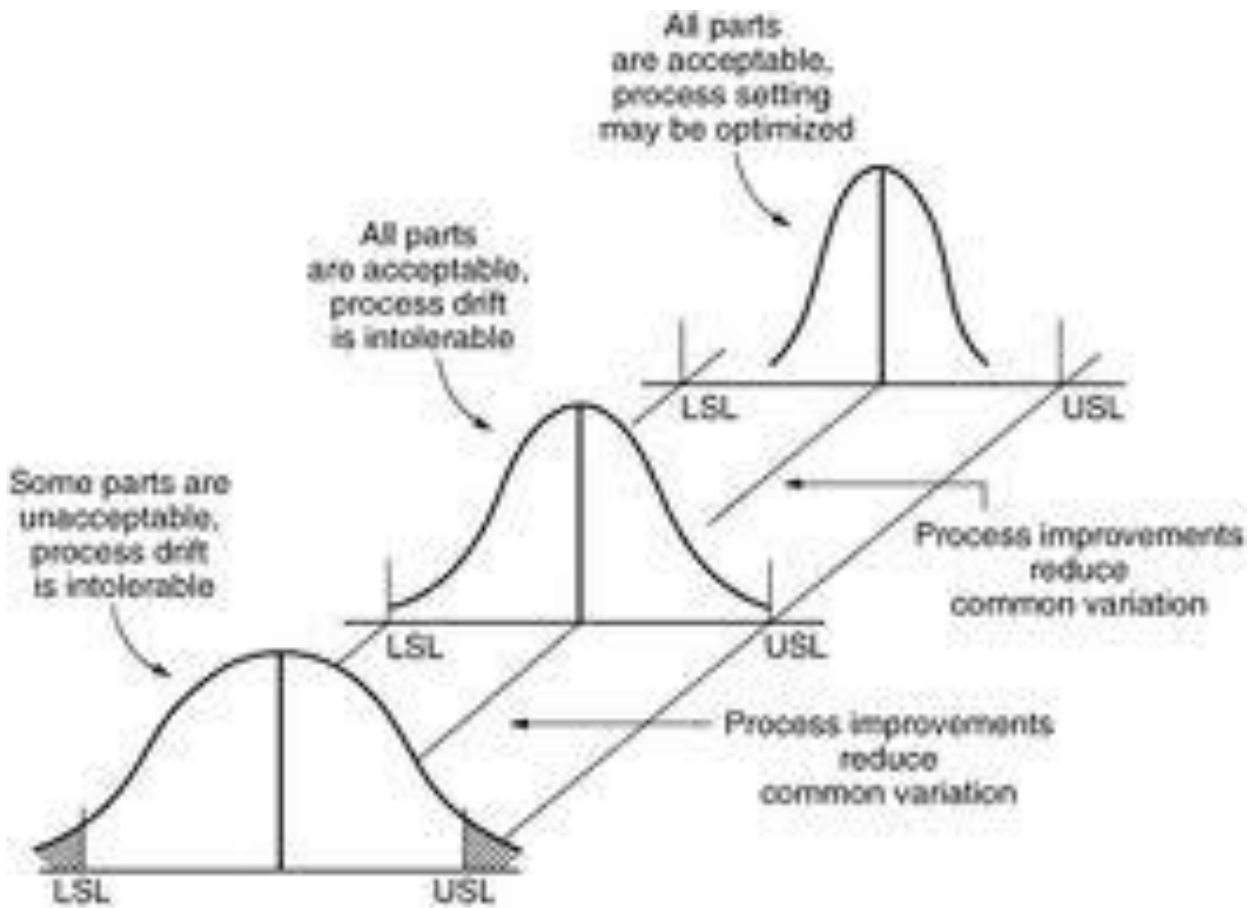
Control Charts
- Process is within statistical control charts

To control, you have to measure!

Production Data always has some Variability



Chance and Assignable Causes of Quality Variation



Accuracy and Precision

Examples of quality characteristics: Painted surface, thickness, hardness, and resistance to fading or chipping, viscosity, sweetness, electrical resistance, frequency, ...

- We can control only those characteristics that can be *counted, evaluated* or *measured*

Engineering characteristics may show problems with accuracy or with precision

Normal Distribution – Shaft Diameter

(What is this plot of data telling us?)

Variability

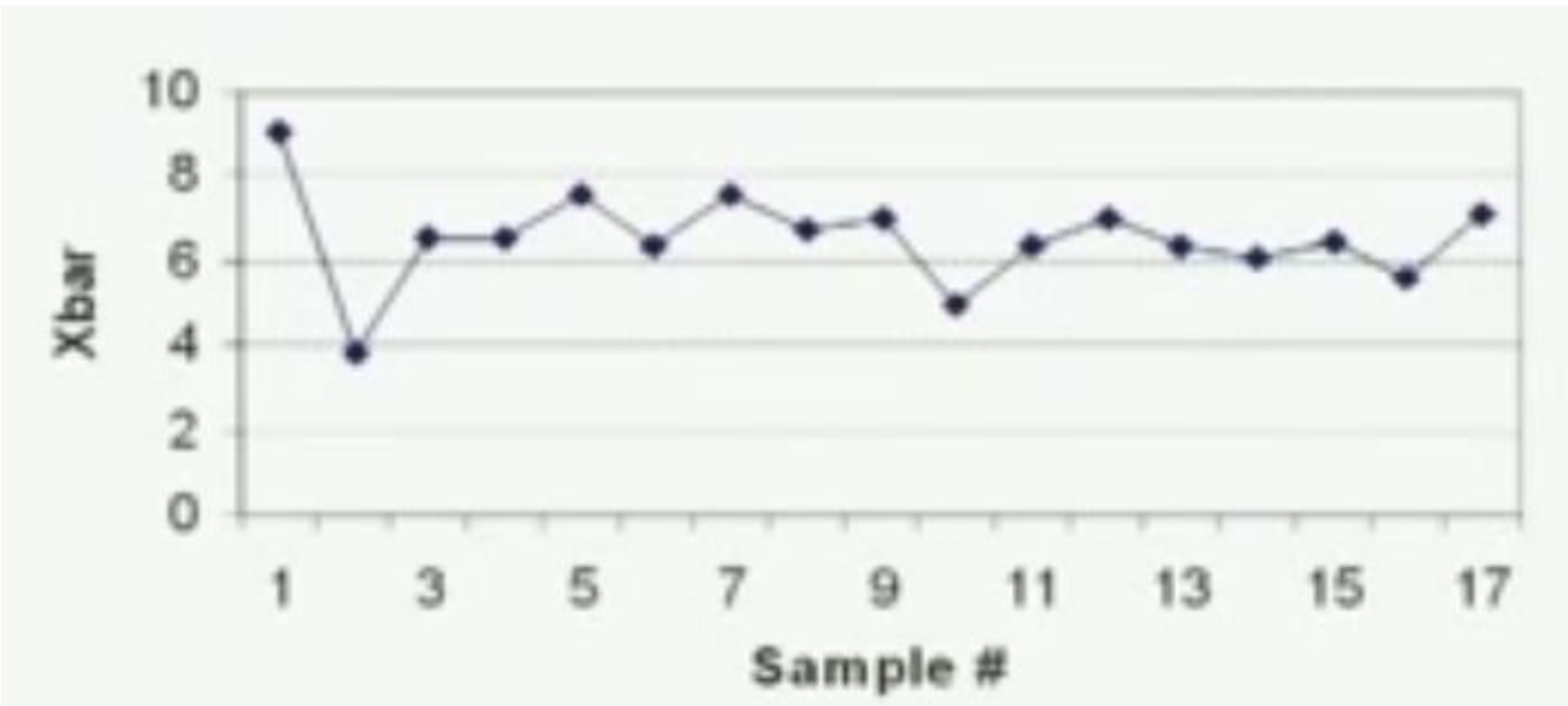
Random

- Common causes
- Inherent in a process
- Can be eliminated only through improvements in the system

Non-Random

- Special causes
- Due to identifiable factors
- Can be modified through operator or management action

Plot of Sample Averages



Control Charts

- A key tool in SPC
- Graph establishing process control limits
- Charts for variables
  Mean (X-bar), Range (R), EWMA, CUSM
- Charts for attributes
  p, np, and c

The Shewhart Control Chart

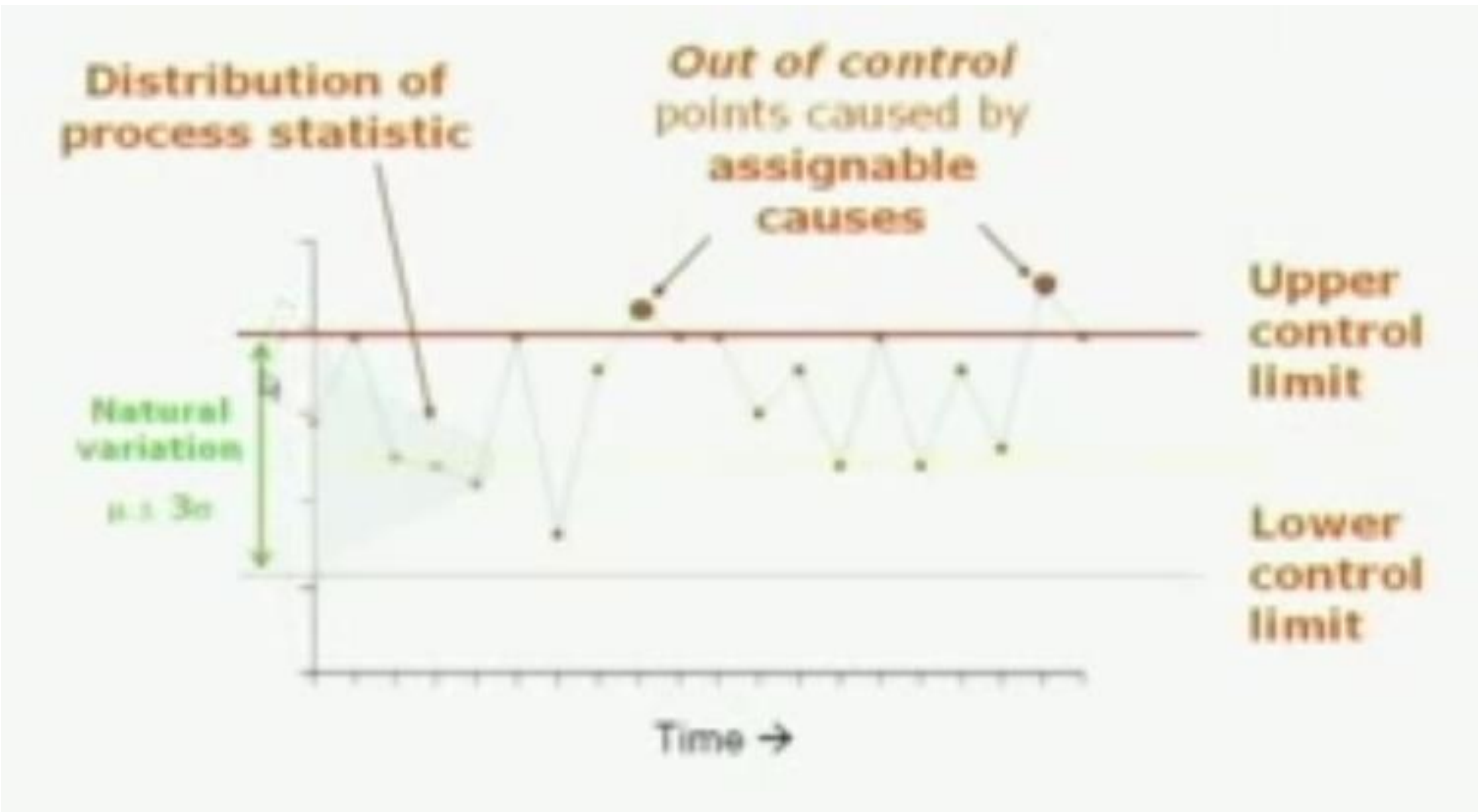
- A time-ordered plot of sample statistics
- When chart is within control limits
  Only random or common causes present
  We leave the process alone

- Plot of each point is the test of hypothesis:

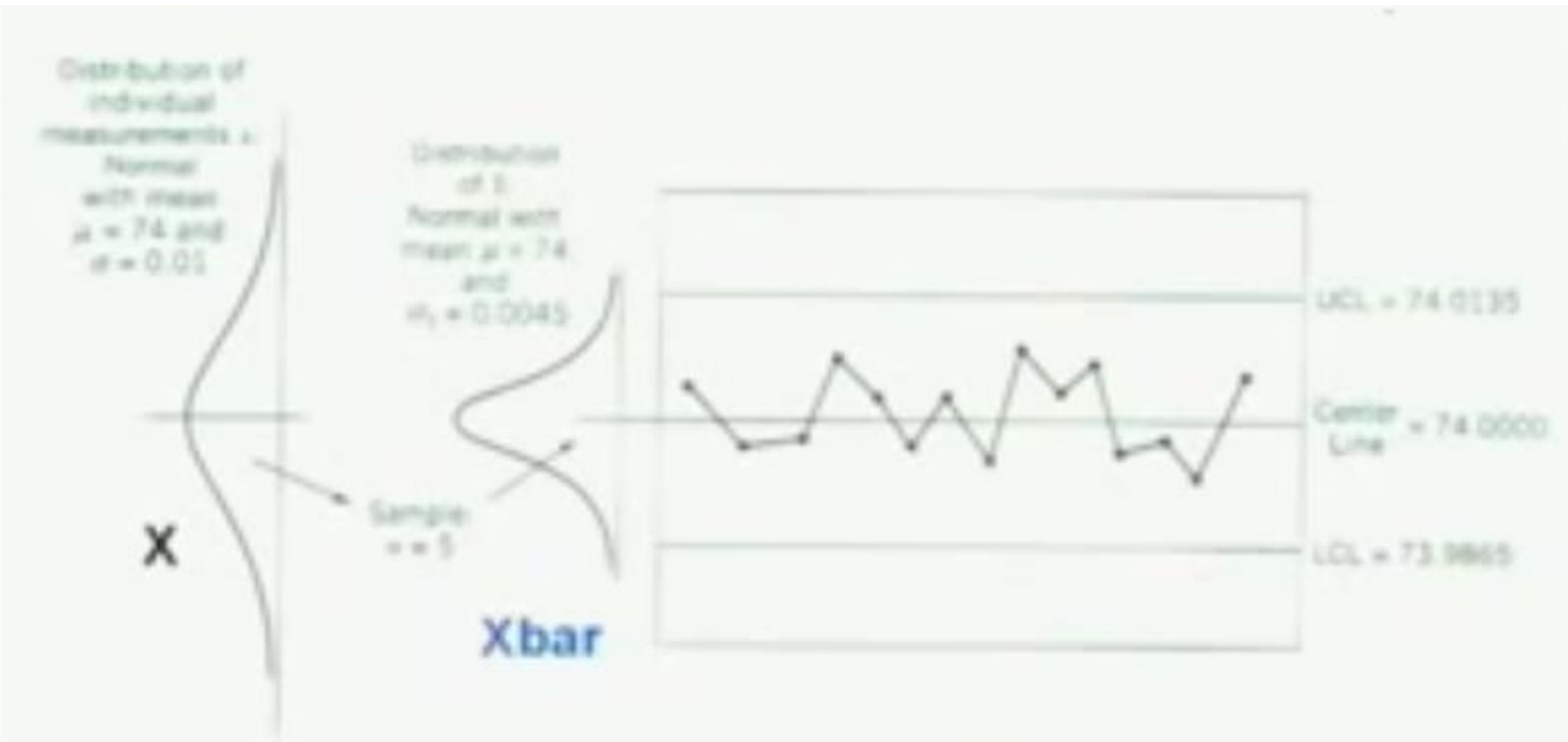
H_0 : Process is “in control” vs.

H_1 : Process is out of control and requires investigation

How Does the Chart Work?



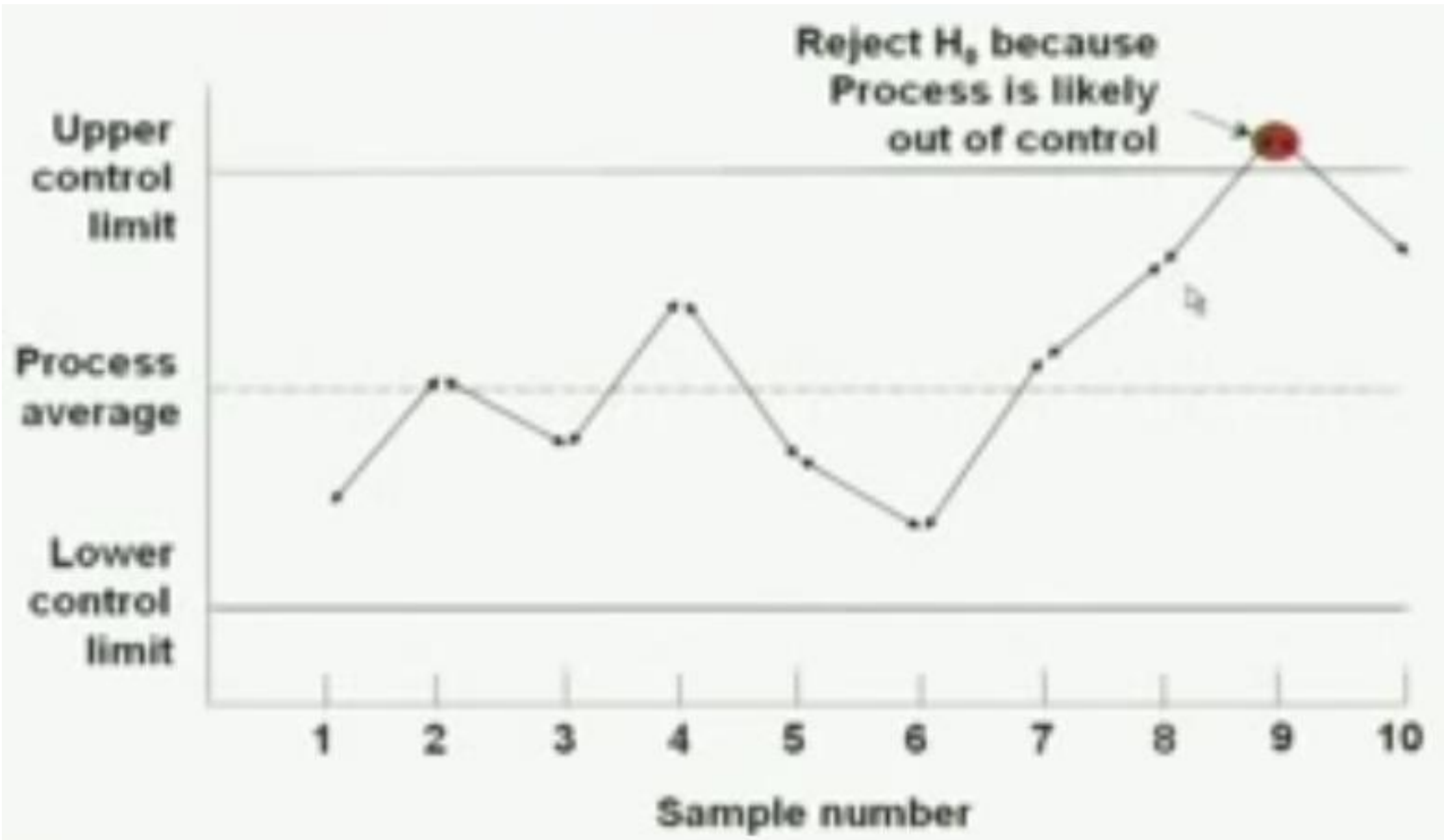
Relationship between “X” (process) and “X-bar” (the control chart)



A Process is “In Control” if

- No sample points are outside limits
- Most points near process average
- About equal number of points above & below centerline
- Points appear randomly distributed
- A process “in control” is supposed to be under the influence of random causes only

The Signal from a Control Chart



Potential Reasons for Variation

- **The Operator:**
  Training, Supervision, Technique.

- **The Method:**
  Procedures, Set-up, Temperature, Cutting Speeds.

- **The Material:**
  Moisture content, Blending, Contamination.

- **The Machine:**
  Set-up, Machine condition, Inherent Precision

Management: Poor process management; poor systems

Charts may signal incorrectly!

Charts repeatedly apply hypothesis testing!

Type I error with charts:

Concluding that a process is not in control when it actually is

Type II error with charts:

Concluding that a process is in control when it is not

Two Types of Process Data

Variables — *"Things we measure"*

- Length
- Weight
- Time
- Blood pressure
- Volume
- Temperature
- Diameter
- Tensile Strength
- Strength of Solution

Attributes — *"Things we count"*

- Number or percent of defective items in a lot.
- Number of defects per item.
- Types of defects.
- Value assigned to defects (minor = 1, major = 5, critical = 10)

Types of Control Charts

- Basic Types

- Most typical three:
  - X-Bar and R
  - p chart
  - c chart

- Depend Upon Data Type:
  - Variables
  - Attribute

- Advanced Types: CUSUM, EWMA, Mulitvariate

- Recall that plotting points on a control chart is the repeated application of Hypothesis Testing

SPC Applied To Services

- Nature of defect is different in services
- Service defect is a failure to meet customer requirement
- Monitor times, customer satisfaction

Applying SPC to Service (cont.)

- **Hospitals**

Timeliness and quickness of care, staff responses to requests, accuracy of lab tests, cleanliness, courtesy, accuracy of paperwork, speed of admittance and checkouts

- **Grocery Stores**

Waiting time to check out, frequency of out-of-stock items, quality of food items, cleanliness, customer complaints, checkout register errors

- **Airlines**

Flight delays, lost luggage and luggage handling, waiting time at ticket counters and check-in, agent and flight attendant courtesy, accurate flight information, passenger cabin cleanliness and maintenance

Applying SPC to Service (cont.)

- **Fast-Food Restaurants**

Waiting time for service, customer complaints, cleanliness, food quality, order accuracy, employee courtesy

- **Catalogue-Order Companies**

Order accuracy, operator knowledge and courtesy, packaging, delivery time, phone order waiting time

- **Insurance Companies**

Billing accuracy, timeliness of claims processing, agent availability and response time

Service SPC Examples

- Hospitals

Timeliness, responsiveness, accuracy

- Grocery Stores

Check-out time, stocking, cleanliness

- Airlines

Luggage handling, waiting times, courtesy

- Fast-food Restaurants

Waiting times, food quality, cleanliness

- Banks

Daily balance errors, # of customers served, transactions completed, courtesy

Types of Shewhart Control Charts

Control Charts for Variables Data

$\bar{X}$ and R charts: for sample averages and ranges.

$\bar{X}$ and s charts: for sample means and standard deviations.

Md and R charts: for sample medians and ranges.

$\bar{X}$ charts: for individual measures; uses moving ranges.

Control Charts for Attributes Data

p charts: proportion of units nonconforming.

np charts: number of units nonconforming.

c charts: number of nonconformities.

u charts: number of nonconformities per unit.

Control Charts for Variables

- Mean chart (X-Bar Chart) ← for accuracy

  Uses average of a sample:

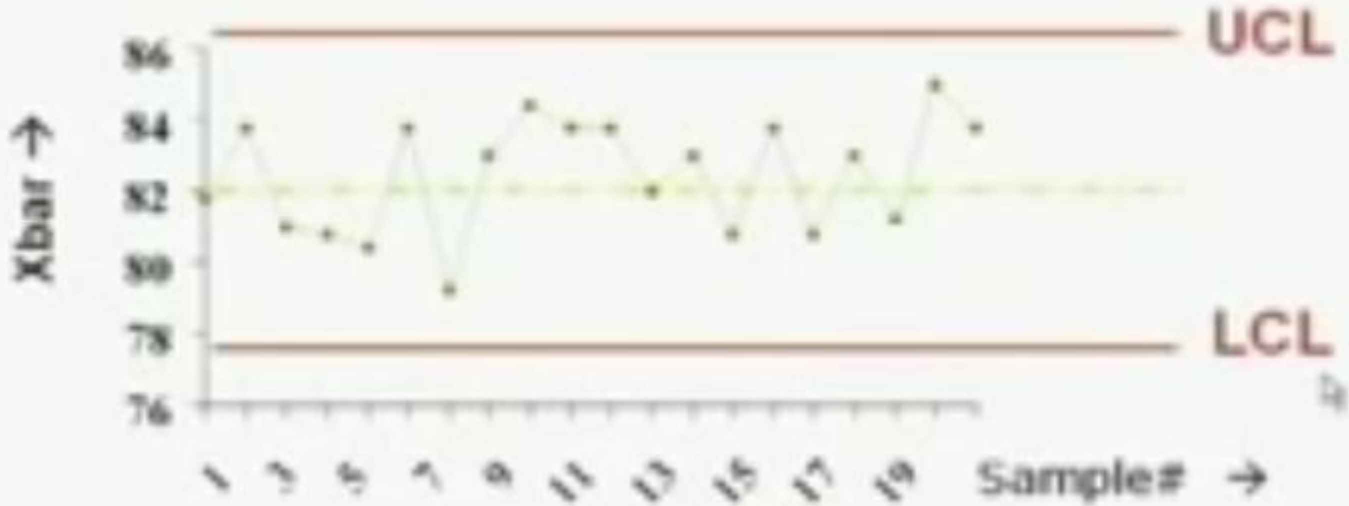
  $$X\text{-Bar} = (x_1+x_2+x_3+x_4+x_5)/5$$

- Range chart (R-Chart) ← for precision

  Uses amount of dispersion in a sample

  $$R = \max(x_i) - \min(x_i)$$

X-bar Chart helps control Accuracy

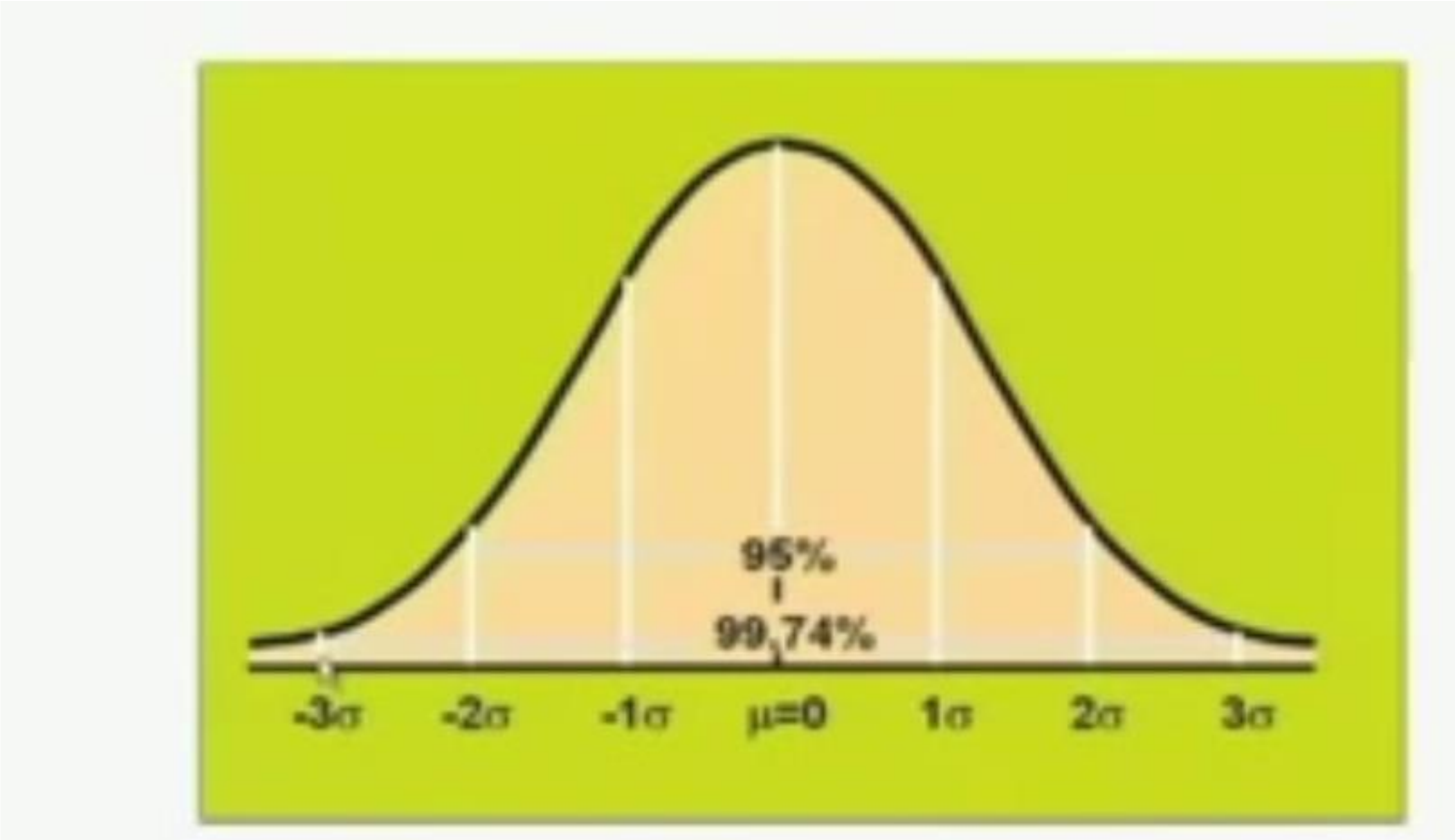


- Average Xbar = 82.5 kg
- Standard Deviation of X bar = σ_{xbar} = 1.6 kg
- Control Limits = Average Xbar $\pm$ 3 σ_{xbar}

  $= 82.5 \pm 3 \cdot 1.6 = [77.7, \ 87.3]$

Here, the process is "in control" (i.e., the mean is stable)

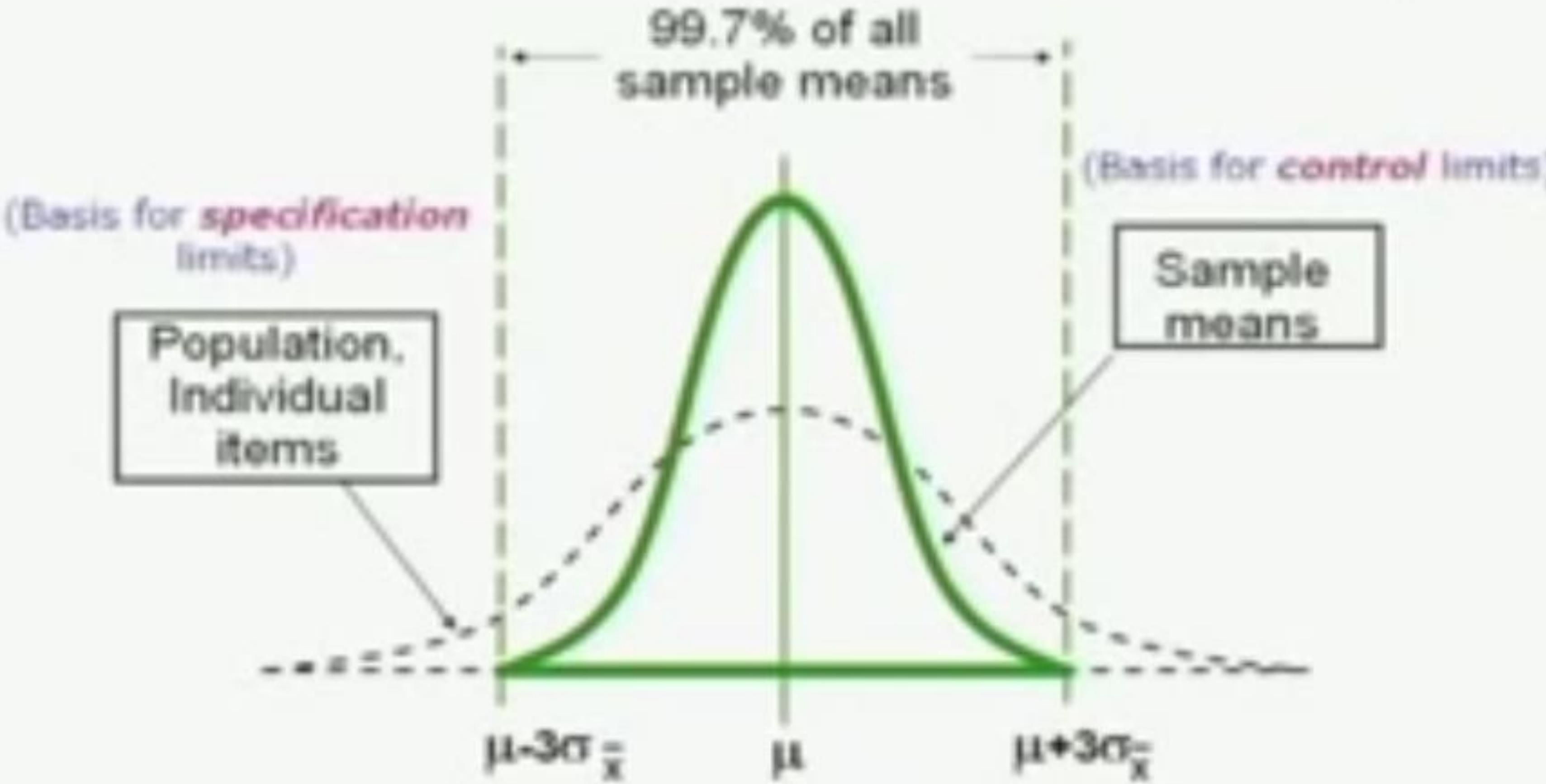
Normal Distribution



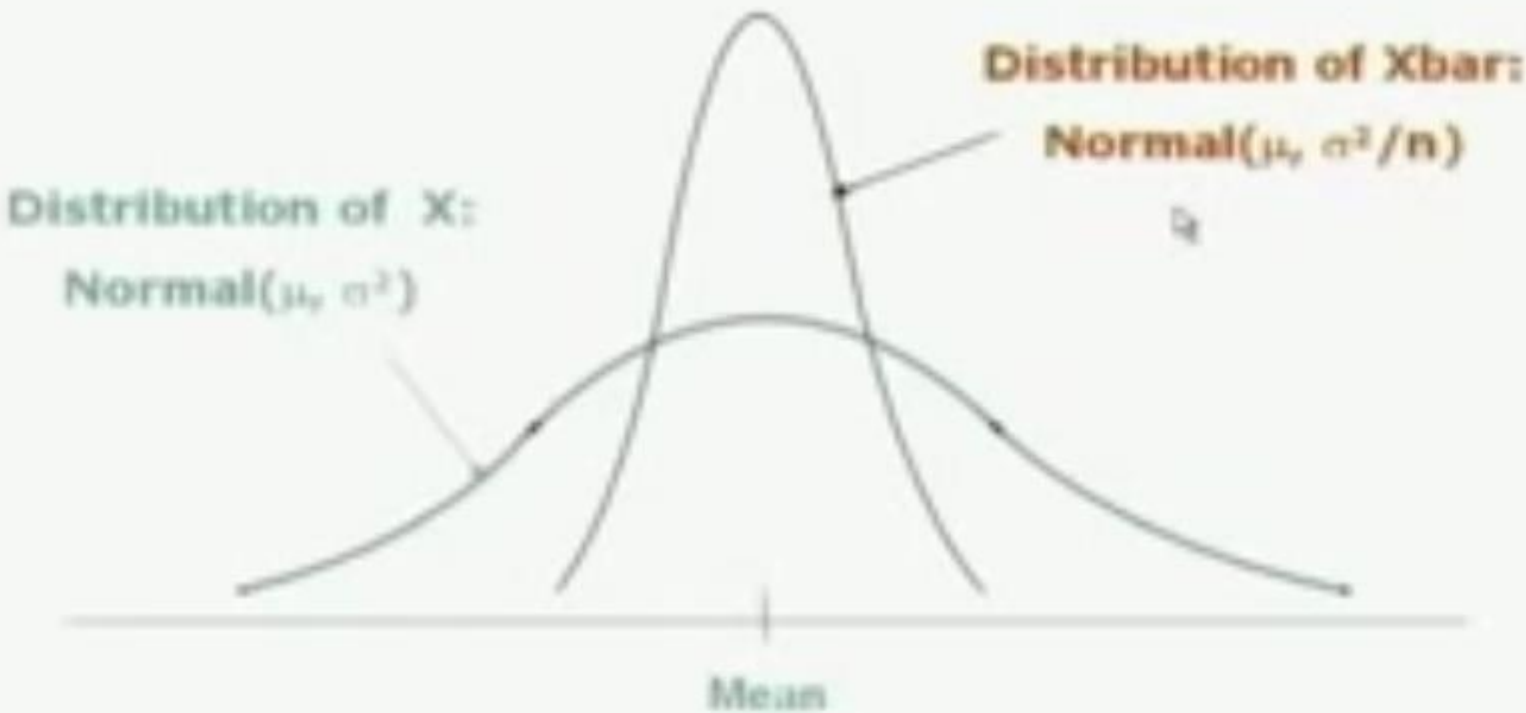
A Process Is in Control If …

1. … no sample points outside limits
2. … most points near process average
3. … about equal number of points above and below centerline
4. … points appear randomly distributed

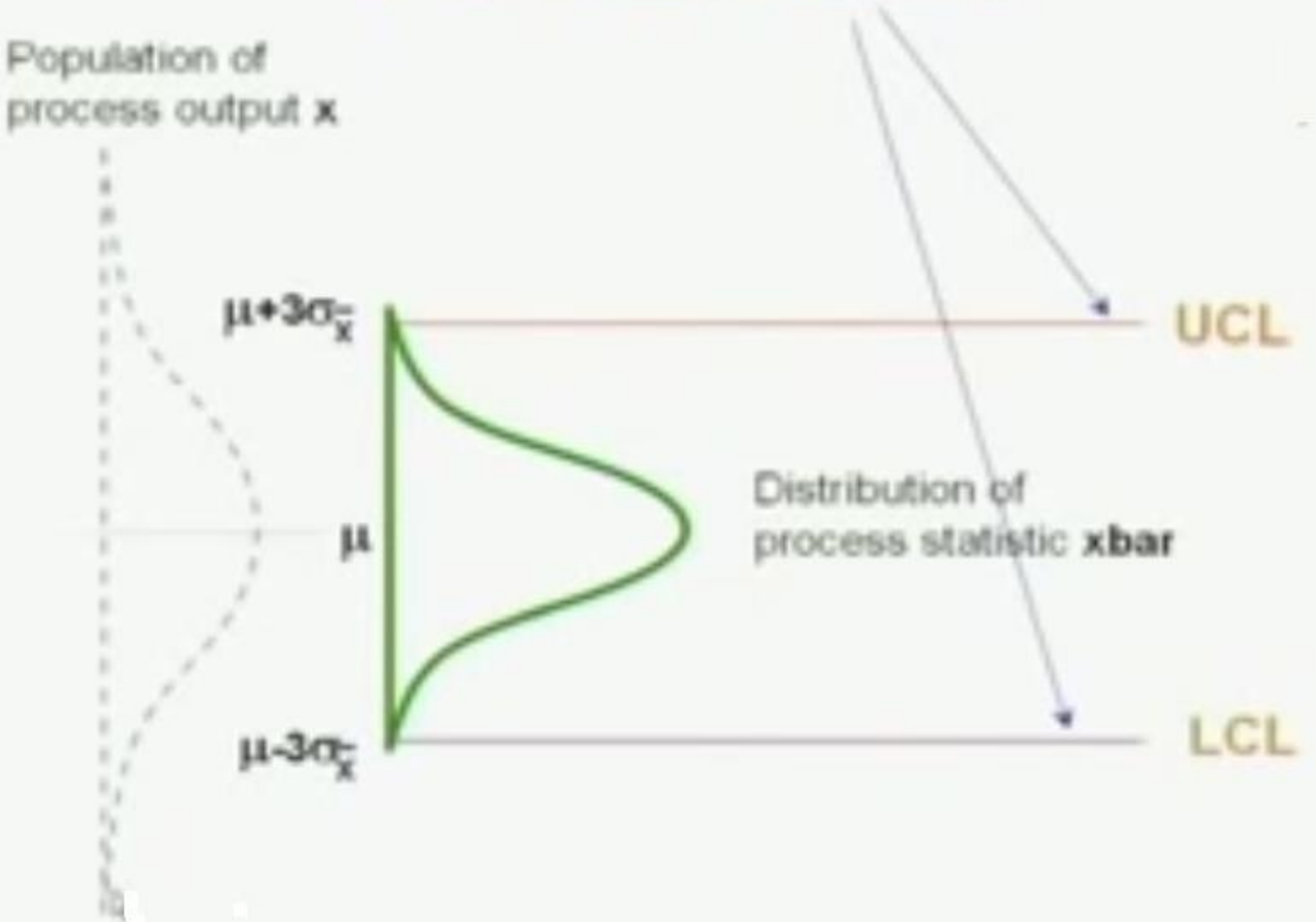
Central Limit Theorem



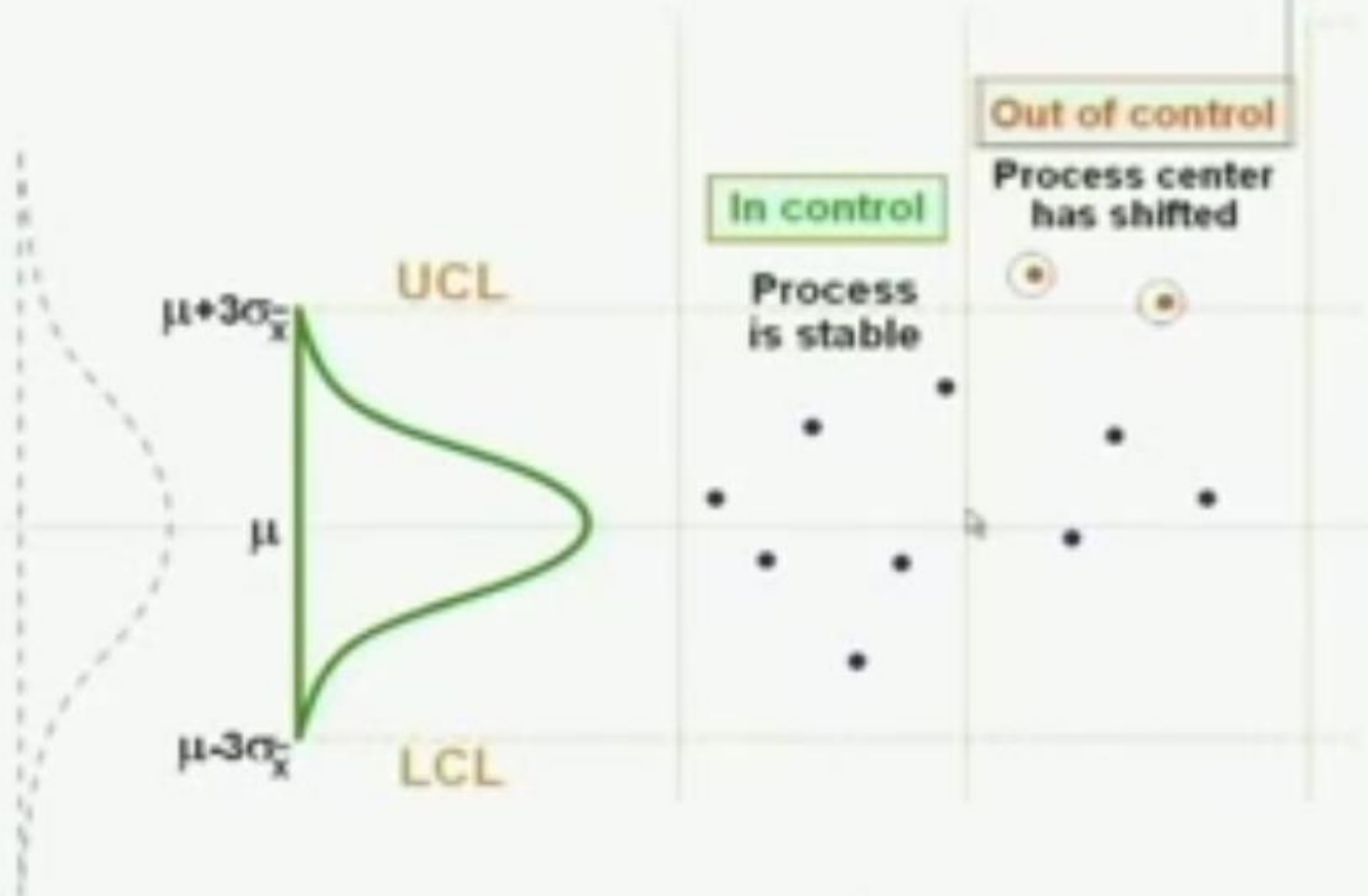
Distribution of Xbar--a Process Statistic



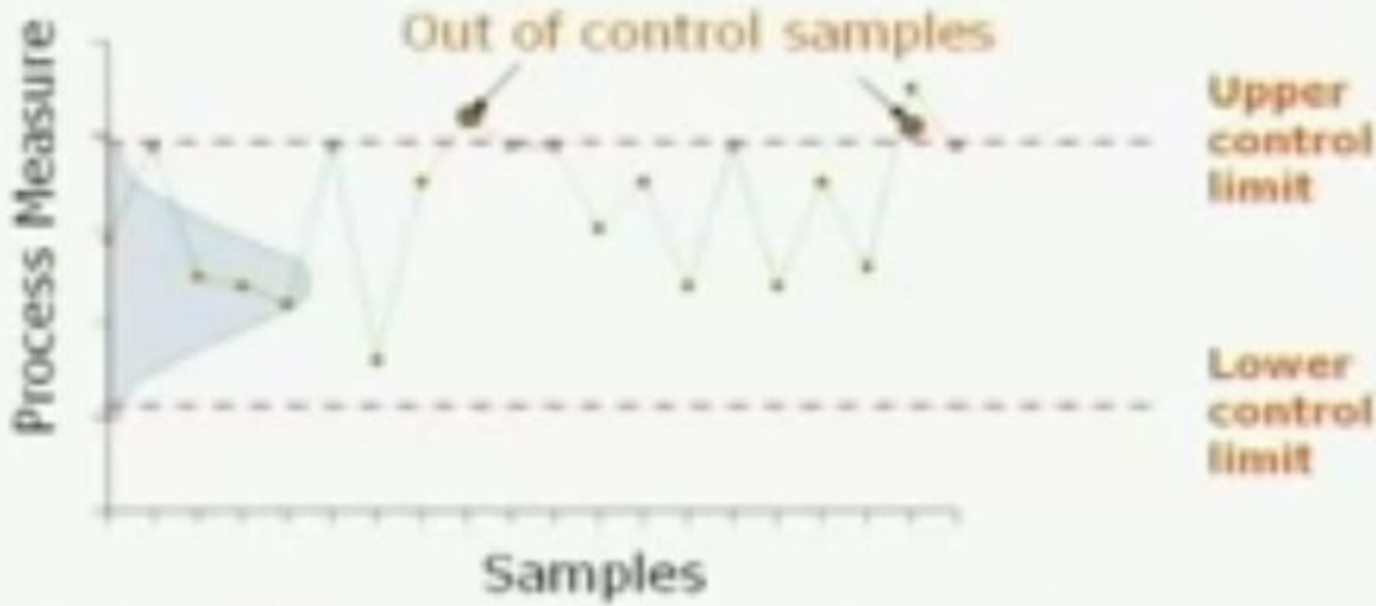
SPC Control Limits



Process Control by Control Limits



Routine use of the Process Control Chart



- Data/Information: Monitor process variability over time
- Control Limits: Average + z · Normal Variability
- Decision Rule:
  - Ignore variability when points are within limits
  - Investigate variation when outside as "abnormal"
- Errors: **Type I** - False alarm (unnecessary investigation)

  Type II - Missed signal (to identify and correct)

Control Charts

- Basic Types
  - Most typical three
    - X-Bar and R
    - p chart
    - c chart
  - Depend Upon Data Type
    - Variables
    - Attribute
- All are Applications of ***Hypothesis Testing:***

  H_0: Process is in statistical control; H_1: Process is out of control

Variations and Control

Random or Common Variation:

Natural or inherent variations in the output of process are created by countless minor factors, too many to investigate economically

Assignable or Special Variation:

A variation whose **cause can be identified**

- Assignable variations push the charts beyond control limits
- Their causes must be investigated, detected and removed

Assignable cause examples: Tool wear, equipment that needs adjustment, defective materials, human factors (carelessness, fatigue, noise and other distractions, failure to follow correct procedures), failure of pumps, heaters, etc.

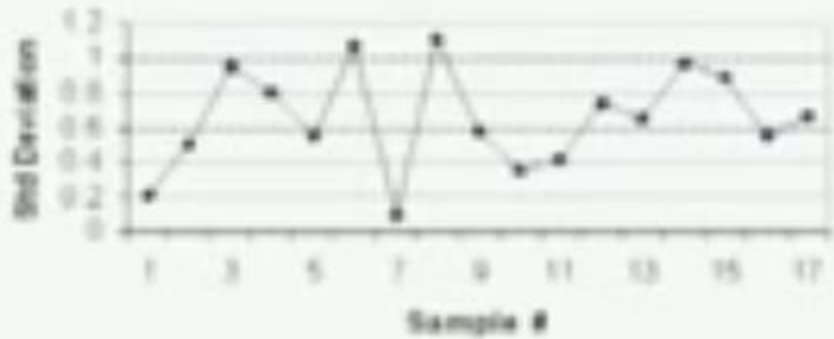
Special Causes of Variation

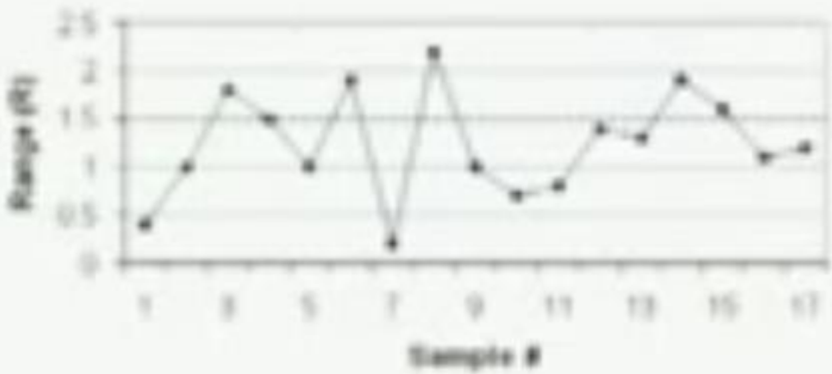
- Also called assignable cause of variation
- When an assignable cause is active, the chart goes beyond control limits
- In SPC, when some unusual or external cause occurs, the cause is identified and data point removed to calculate true control limits
- Attempting to improve a process (containing special cause variation) without removing the special cause only increases the instability and variation of the process

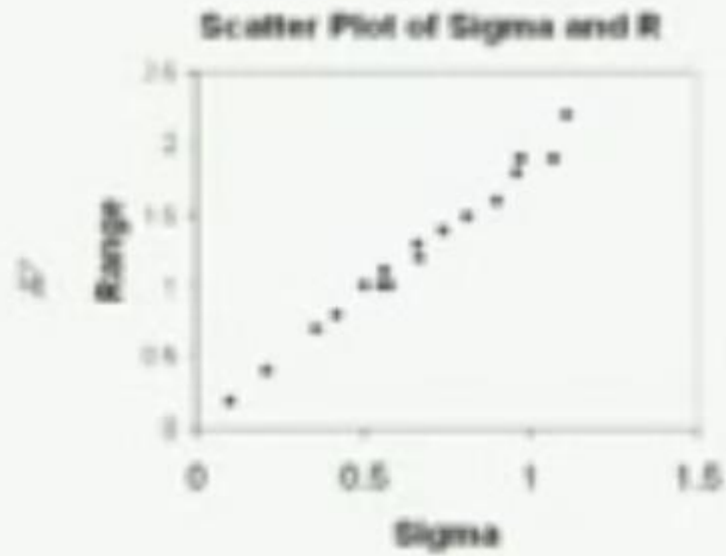
Common Causes of Variation

- Also called random causes of variation
- When only common causes are active, the chart remains stable and within control limits
- In SPC, when only random causes are active, no single cause is at fault. Any process improvement effort now must consider all sources of variation, generally the factors inherent in the technology of the process
- A process with only common cause of variation is stable and predictable and it forms the basis for measuring process capability

We can use Range in place of Std Deviation to control Precision







Correlation(s, R) = 0.9934

Control Charts for Variables

- Mean chart (X-Bar Chart)
  - Uses average of a sample
- Range chart (R-Chart)
  - Uses amount of dispersion in a sample

Construction of Control Chart

- Control limits must be based only on historic process data that are "in-control"
- We draw tentative limit lines and check if any points fall outside the limits
- If some points fall outside, non-random causes are present; discard those data points and re-calculate control limits
- Repeat calculation of limits if necessary

x-bar Chart

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \ldots \bar{x}_k}{k}$$

$$UCL = \bar{\bar{x}} + A_2\bar{R} \qquad LCL = \bar{\bar{x}} - A_2\bar{R}$$

where

$\bar{\bar{x}}$ = average of sample means

x-bar Chart Example

	OBSERVATIONS (SLIP-RING DIAMETER, CM)						
SAMPLE *k*	*1*	*2*	*3*	*4*	*5*	$\bar{x}$	*R*
1	5.02	5.01	4.94	4.99	4.96	4.98	0.08
2	5.01	5.03	5.07	4.95	4.96	5.00	0.12
3	4.99	5.00	4.93	4.92	4.99	4.97	0.08
4	5.03	4.91	5.01	4.98	4.89	4.96	0.14
5	4.95	4.92	5.03	5.05	5.01	4.99	0.13
6	4.97	5.06	5.06	4.96	5.03	5.01	0.10
7	5.05	5.01	5.10	4.96	4.99	5.02	0.14
8	5.09	5.10	5.00	4.99	5.08	5.05	0.11
9	5.14	5.10	4.99	5.08	5.09	5.08	0.15
10	5.01	4.98	5.08	5.07	4.99	5.03	0.10
						50.09	1.15

x- bar Chart Example (cont.)

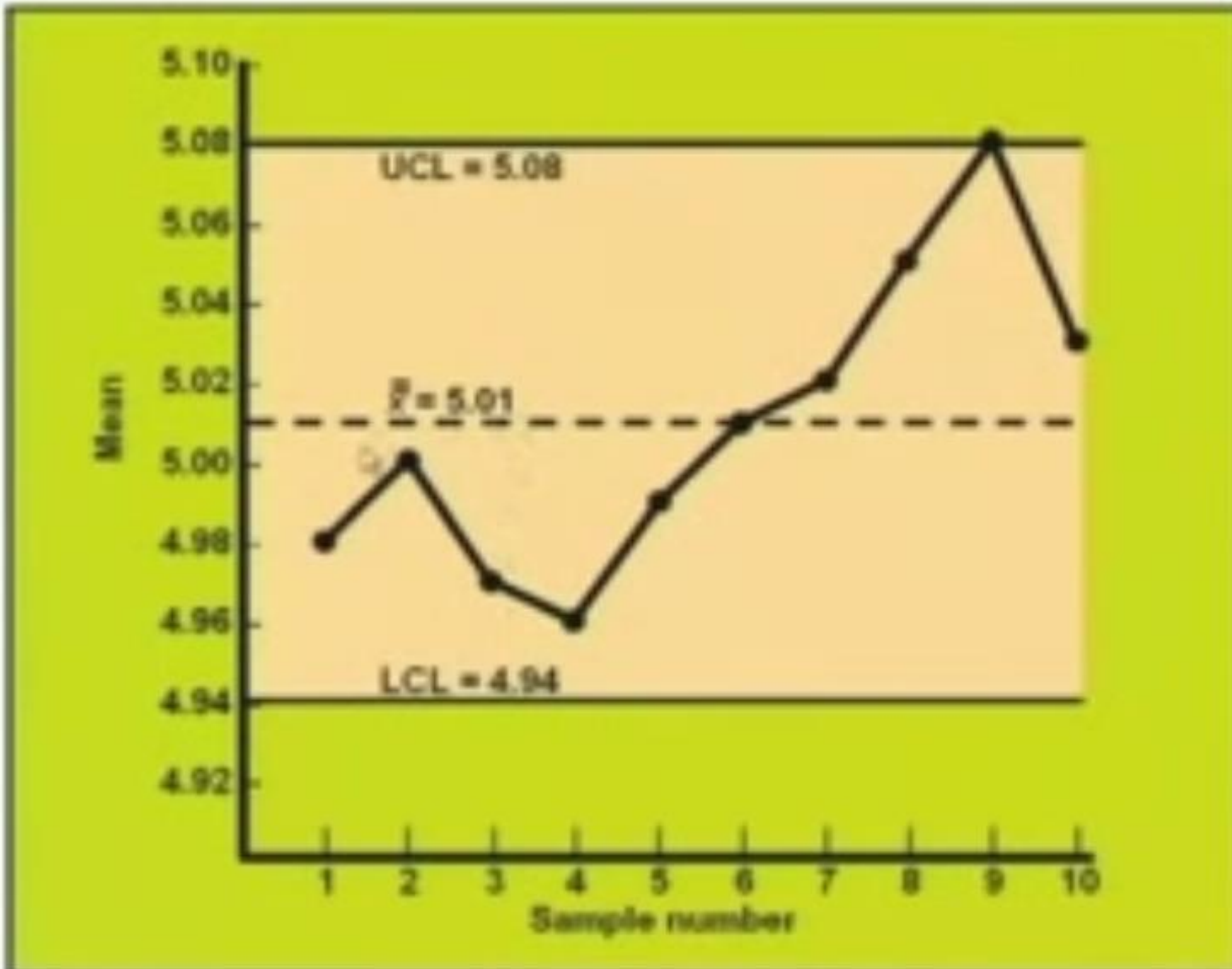
$$\bar{\bar{x}} = \frac{\sum \bar{x}}{k} = \frac{50.09}{10} = 5.01 \text{ cm}$$

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 5.01 + (0.58)(0.115) = 5.08$$

$$LCL = \bar{x} - A_2\bar{R} = 5.01 - (0.58)(0.115) = 4.94$$

Retrieve Factor Value A_2

x- bar
Chart
Example
(cont.)



Three Sigma Control Limits

- The use of 3-sigma limits generally gives good results in practice (ARL = $1/(\alpha/2)$).
- If the distribution of the quality characteristic is reasonably well approximated by the normal distribution, then the use of 3-sigma limits is applicable.
- These limits are often referred to as action limits.

Warning Limits on Control Charts

- Warning limits (if used) are typically set at 2 standard deviations from the mean.
- If one or more points fall between the warning limits and the control limits, or close to the warning limits the process may not be operating properly.
- Good thing: Warning limits often increase the *sensitivity* of the control chart.
- Bad thing: Warning limits could result in an increased risk of false alarms.

Control Charts for Variables

- Mean chart (X-Bar Chart)
  - Uses average of a sample
- Range chart (R-Chart)
  - Uses amount of dispersion in a sample

Calculation of Xbar Chart's Control Limits

Def : Xbar $= \bar{x} = (x_1 + x_2 + x_3 + x_4 + x_5)/5$

Range $R = Max\ x_i - Min\ x_i$

A quick method for finding control limits is to use

average sample range $\bar{R}$ as a measure of process variability.

Upper control limit. UCL $= \bar{\bar{x}} + z\sigma_{xbar} = \bar{\bar{x}} + A_2\bar{R}$

Lower control limit. LCL $= \bar{\bar{x}} - z\sigma_{xbar} = \bar{\bar{x}} - A_2\bar{R}$

where $\bar{R}$ = Average of sample ranges and A_2 is found from a table.

Process Control Chart Factors

Sample (Subgroup) Size (n)	Control Limit Factor for Averages (Mean Charts) (A_2)	UCL Factor for Ranges (Range Charts) (D_4)	LCL Factor for Ranges (Range Charts) (D_3)	Factor for Estimating Sigma ($\sigma = R/d_2$) (d_2)
2	1.880	3.267	0	1.128
3	1.023	2.575	0	1.693
4	0.729	2.282	0	2.059
5	0.577	2.115	0	2.326
6	0.483	2.004	0	2.534
7	0.419	1.924	0.076	2.704
8	0.373	1.864	0.136	2.847
9	0.337	1.816	0.184	2.970
10	0.308	1.777	0.223	3.078

Process Data Example:

Select 25 small samples (in this case, n = 4)

Find $\bar{X}$ and R of each sample.

The $\bar{\bar{X}}$ chart is used to control the process mean.

The R chart is used to control process variation.

	Sample Number					
	1	2	3	4	25	
X Values	4	7	6	7		
	6	3	9	6		
	5	8	8	6		
	5	6	9	5		
Sum	20	24	32	24	28	Total
$\bar{X}$	5	6	8	6	7	150
R	2	5	3	2	3	75

$\bar{X}$ and R Chart Plots

n	A_2	D_3	D_4	d_2
2	1.880	3.267	0	1.128
3	1.023	2.575	0	1.693
4	0.729	2.282	0	2.059

Sample Number

	1	2	3	4	…	25	
Values	4	7	6	7			
	6	3	9	6			
	5	8	8	6			
	5	6	9	5			
Sum	20	24	32	24		28	Total
X	5	6	8	6		7	150
R	2	5	3	2		3	75

$\bar{\bar{X}} = 150 / 25 = 6$

$\bar{R} = 75 / 25 = 3$

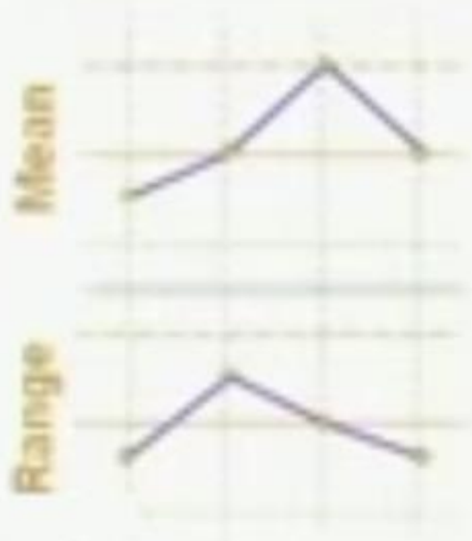
$A_2\bar{R} = 0.729(3) = 2.2$

$UCL_{\bar{x}} = \bar{\bar{X}} + A_2\bar{R} = 6 + 2.2 = 8.2$

$LCL_{\bar{x}} = \bar{\bar{X}} - A_2\bar{R} = 6 - 2.2 = 3.8$

$UCL_R = D_4\bar{R} = 2.282(3) = 6.8$

$LCL_R = D_3\bar{R} = 0(3) = 0$



$UCL_{\bar{x}} = 8.2$

$\bar{\bar{X}} = 6.0$

$LCL_{\bar{x}} = 3.8$

$UCL_R = 6.8$

$\bar{R} = 3.0$

$LCL_R = 0$

Example: Xbar chart Control Limits by σ_{xbar}

A quality control manager took five samples (S1, S2, S3, S4, S5), each with four observations, of the diameter of shafts manufactured on a lathe machine. The manager computed the mean of each sample and then computed the grand mean. All values are in cm. Use this information to obtain 3-sigma (i.e., z=3) control limits for means of future times. It is known from previous experience that the **standard deviation** σ_x of the process is 0.02 cm.

Observation	S1	S2	S3	S4	S5
1	12.11	12.15	12.09	12.12	12.09
2	12.10	12.12	12.09	12.10	12.14
3	12.11	12.10	12.11	12.08	12.13
4	12.08	12.11	12.15	12.10	12.12
Xbar	12.10	12.12	12.11	12.10	12.12

R- Chart

$$UCL = D_4\bar{R} \qquad LCL = D_3\bar{R}$$

$$\bar{R} = \frac{\sum R}{k}$$

where

$\bar{R}$ = range of each sample
k = number of samples

R-Chart Example

SAMPLE k	OBSERVATIONS (SLIP-RING DIAMETER, CM)						
	1	*2*	*3*	*4*	*5*	$\bar{x}$	*R*
1	5.02	5.01	4.94	4.99	4.96	4.98	0.08
2	5.01	5.03	5.07	4.95	4.96	5.00	0.12
3	4.99	5.00	4.93	4.92	4.99	4.97	0.08
4	5.03	4.91	5.01	4.98	4.89	4.96	0.14
5	4.95	4.92	5.03	5.05	5.01	4.99	0.13
6	4.97	5.06	5.06	4.96	5.03	5.01	0.10
7	5.05	5.01	5.10	4.96	4.99	5.02	0.14
8	5.09	5.10	5.00	4.99	5.08	5.05	0.11
9	5.14	5.10	4.99	5.08	5.09	5.08	0.15
10	5.01	4.98	5.08	5.07	4.99	5.03	0.10
						50.09	1.15

R-Chart Example (cont.)

$$\bar{R} = \frac{\sum R}{k} = \frac{1.15}{10} = 0.115$$

$$UCL = D_4\bar{R} = 2.11(0.115) = 0.243$$

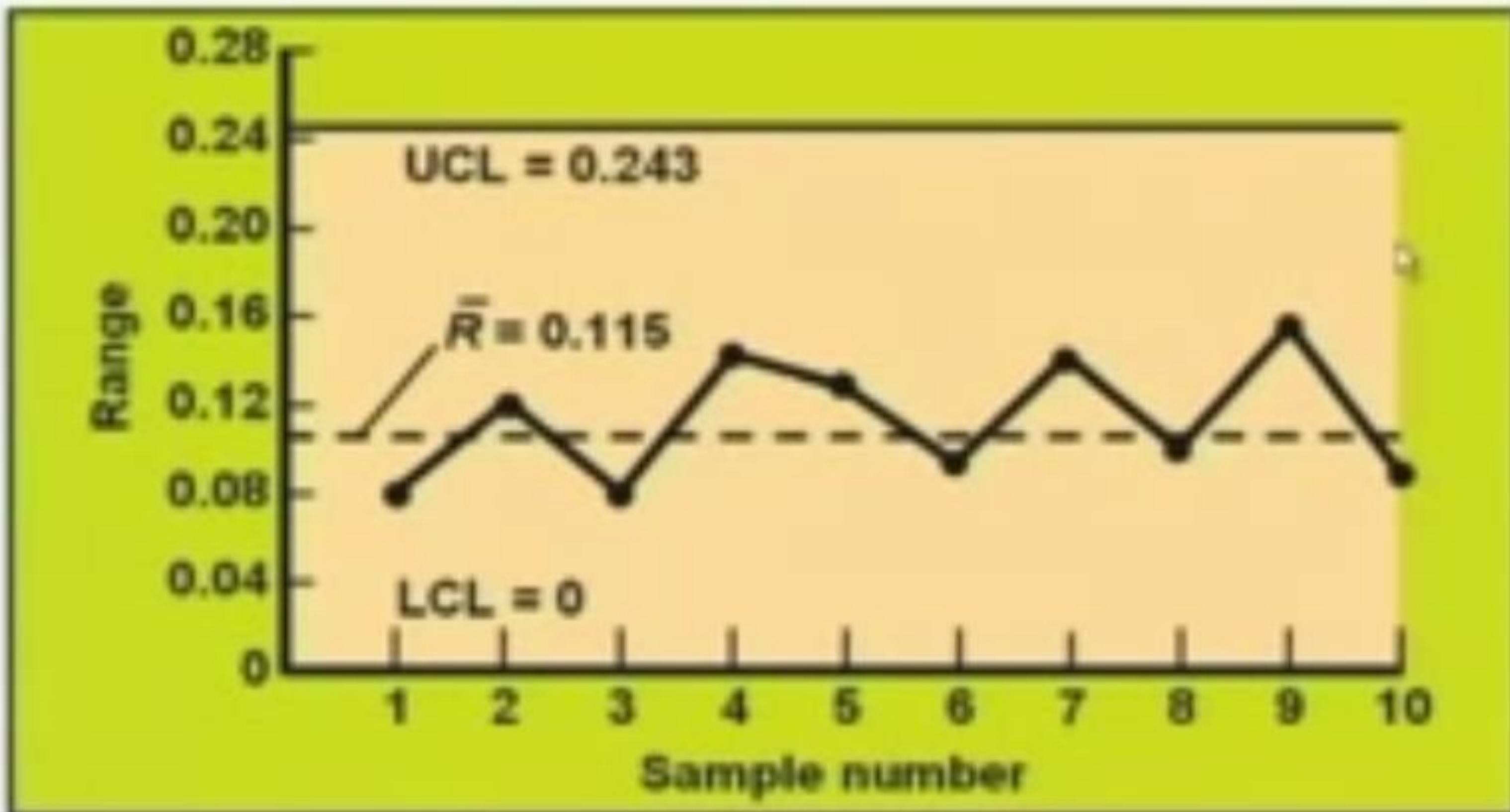
$$LCL = D_3\bar{R} = 0(0.115) = 0$$

Retrieve Factor Values D_3 and D_4

Control Limit Factors

Sample Size	Factor for Xbar limits	Factor for R LCL	Factor for R UCL
n	A_2	D_3	D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65
16	0.21	0.36	1.64
17	0.20	0.38	1.62
18	0.19	0.39	1.61
19	0.19	0.40	1.60
20	0.18	0.41	1.59

R-Chart Example (cont.)



Example: R chart Limits

Observation	S1	S2	S3	S4	S5
1	12.11	12.15	12.09	12.12	12.09
2	12.10	12.12	12.09	12.10	12.14
3	12.11	12.10	12.11	12.08	12.13
4	12.08	12.11	12.15	12.10	12.12
Xbar	12.10	12.12	12.11	12.10	12.12
Range *R*	0.03	0.05	0.06	0.04	0.05

$\bar{R}$ = Average of sample ranges

$$= \frac{0.03 + 0.05 + 0.06 + 0.04 + 0.05}{5} = 0.046$$

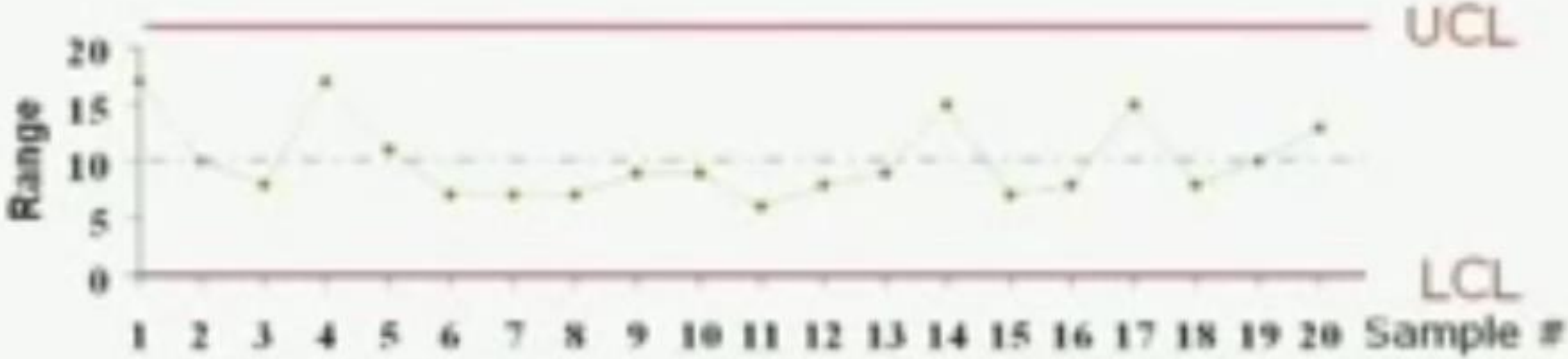
$n = 4$ Therefore $D_3 = 0.00$ and $D_4 = 2.28$ from table

Hence, *Upper / Lower Control Limits* are

$$UCL_R = D_4\bar{R} = 2.280 \cdot .046 = 0.105$$

$$LCL_R = D_3\bar{R} = 0.000 \cdot .046 = 0.00$$

Range (R) Chart helps control *Precision*



- Average Range *R* = 10.1 kg
- Standard Deviation of Range = 3.5 kg
- Control Limits: 10.1 ± 3 3.5 = [20.6, 0]
  - Process here is "in control" (i.e., precision is stable)

Example: R chart Limits

Observation	S1	S2	S3	S4	S5
1	12.11	12.15	12.09	12.12	12.09
2	12.10	12.12	12.09	12.10	12.14
3	12.11	12.10	12.11	12.08	12.13
4	12.08	12.11	12.15	12.10	12.12
Xbar	12.10	12.12	12.11	12.10	12.12
Range *R*	0.03	0.05	0.06	0.04	0.05

$\bar{R}$ = Average of sample ranges

$$= \frac{0.03 + 0.05 + 0.06 + 0.04 + 0.05}{5} = 0.046$$

$n = 4$ Therefore $D_3 = 0.00$ and $D_4 = 2.28$ from table

Hence, *Upper / Lower Control Limits* are

$$UCL_R = D_4\bar{R} = 2.280 \times .046 = 0.105$$

$$LCL_R = D_3\bar{R} = 0.000 \times .046 = 0.00$$

Monitoring the Process by Control Charts

We look now for abnormal patterns on the charts

Using x-bar and R-Charts Together

- Process average and process variability must be in control.

- It is possible for samples to have very narrow ranges, but their averages are beyond control limits.

- It is possible for sample averages to be in control, but ranges might be very large.

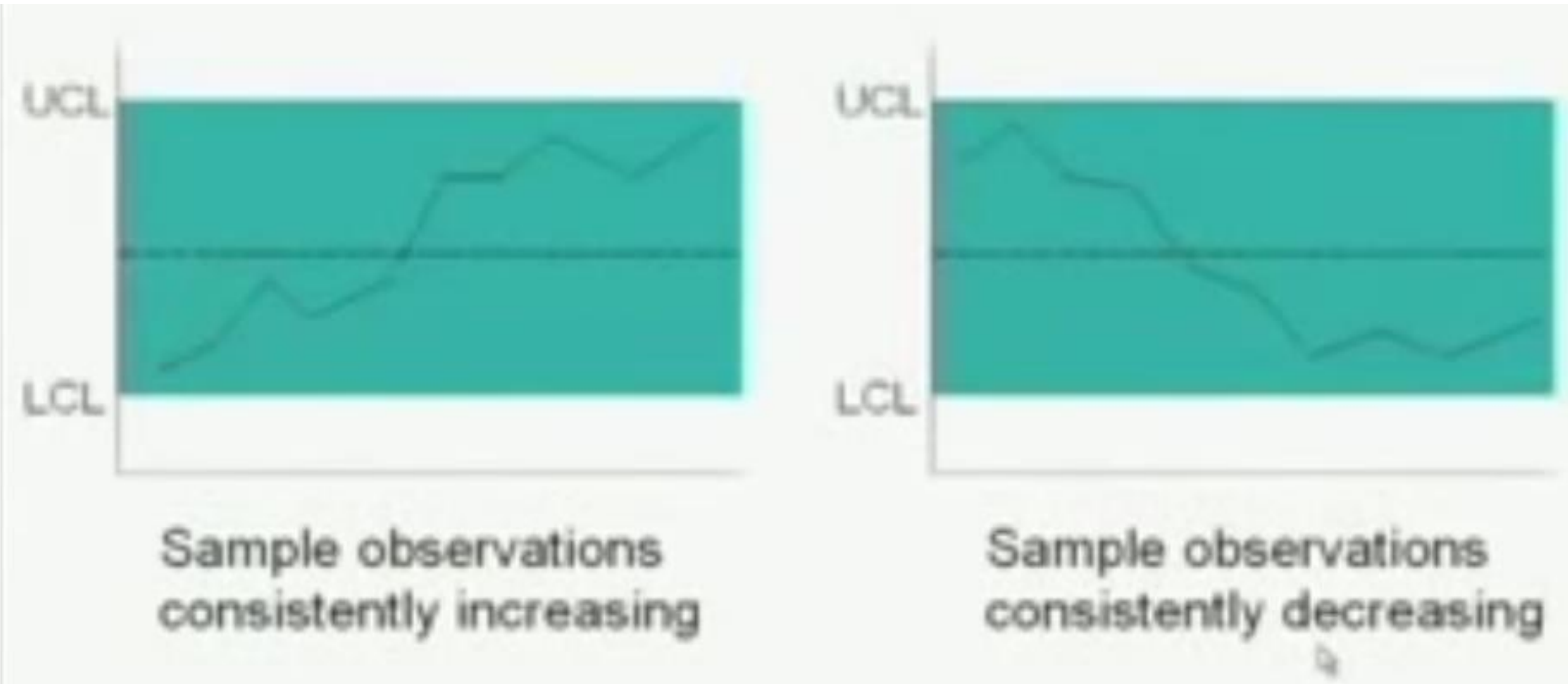
Performance Variation Patterns

Abnormal Control Chart Patterns

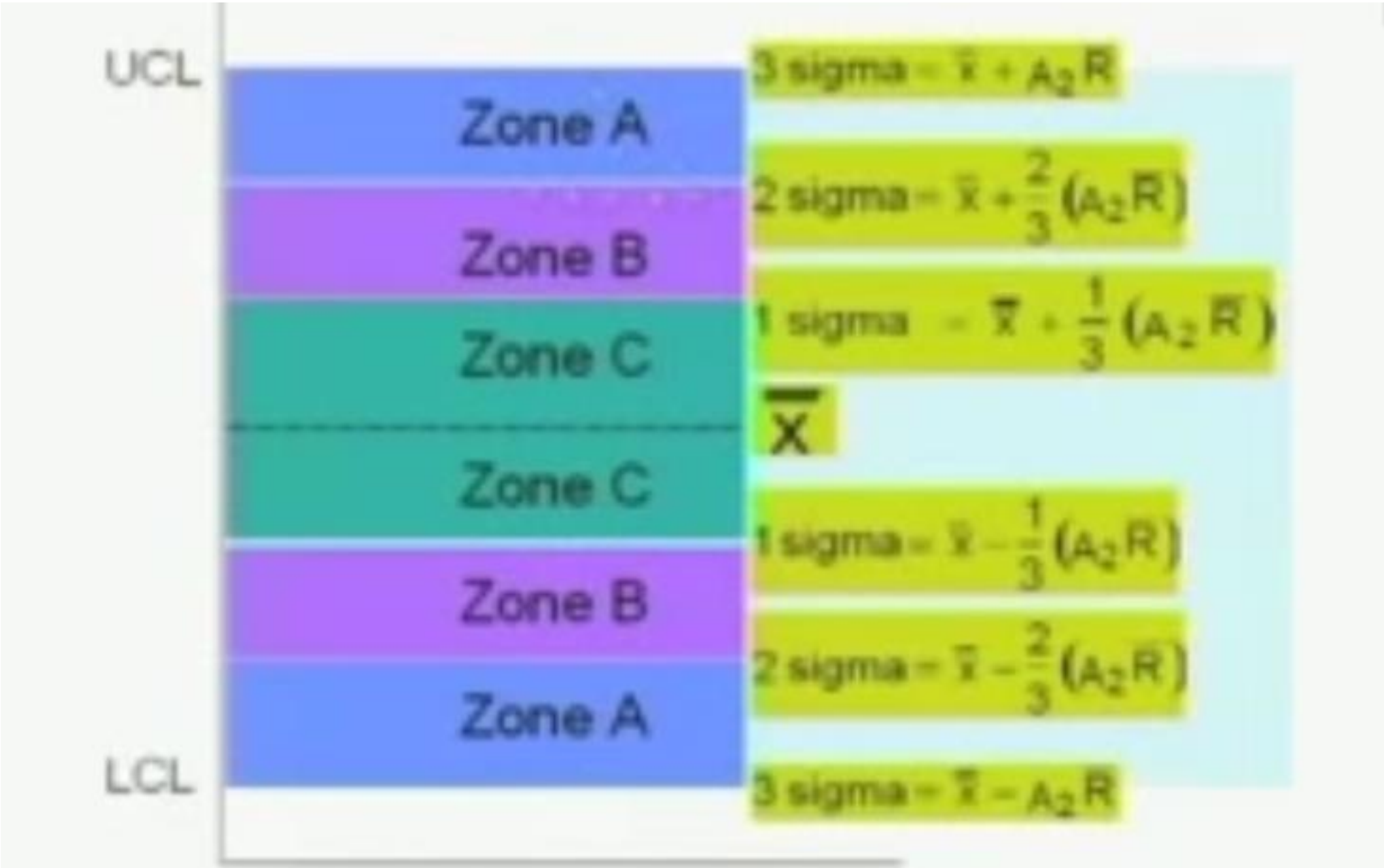
Sample observations consistently below the center line

Sample observations consistently above the center line

Abnormal Control Chart Patterns



Zones for Non-Random Pattern Tests



Abnormal Control Chart Patterns

1. 8 consecutive points on one side of the centre line
2. 8 consecutive points up or down across zones.
3. 14 points alternating up or down
4. 2 out of 3 consecutive points in zone A but still inside the control limits
5. 4 out of 5 consecutive points in zone A or B

Reaction Plan to use when process is out of control

Sample Size

Attribute charts require larger sample sizes: 50 to 100 parts in a sample

Variable charts require smaller samples: 2 to 10 parts in a sample

From Control to Improvement