# The Invertible Matrix Theorem

If we have an $n \times n$ matrix $A$, then the following statements are *equivalent*. This means that if any of these statements is true for a particular $A$ then they are all true and if any is false then they are all false.

1. $A$ is invertible, that is, $A^{-1}$ exists.
2. $A$ is row-equivalent to an $n \times n$ identity matrix. That is, row operations can be used to change $A$ into $I_n$.
3. $A$ has $n$ pivot positions
4. The equation $A\vec{x} = \vec{0}$ has only the trivial solution, that is, $\vec{x} = \vec{0}$.
5. The determinant of $A$ is not zero.
6. The columns of $A$ span $\mathbb{R}^n$.
7. The columns of $A$ form a linearly independent set.
8. The columns of $A$ form a basis of $\mathbb{R}^n$.
9. The equation $A\vec{x} = \vec{b}$ has at least one solution for each $b$ in $\mathbb{R}^n$.
10. The column space of $A$ is $\mathbb{R}^n$.
11. The null space of $A$ is $\{\vec{0}\}$.
12. The rank of $A$ is $n$.
13. The dimension of the column space of $A$ is $n$.
14. The dimension of the null space of $A$ is 0.
15. The number 0 is not an eigenvalue of $A$.
16. $A$ has $n$ nonzero singular values.